INTEGRAL EQUATIONS IN PROBLEMS OF REPRESENTATION OF FUNCTIONS OF A COMPLEX VARIABLE

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1. **Introduction.** A work of the present author [2] contains a number of representations of the form

$$f(z) = \iint_K \frac{d\mu(e_J)}{J - z} - a(z) \qquad (z \text{ in } K); \tag{1.1}$$

here $f(z)$ is a given function (of a suitable class, generally non-analytic) of the variable $z = x + iy$; K is a bounded connected domain in the complex plane, whose frontier $\bar{K} - K$ has zero two-dimensional measure; integration is in Lebesgue-Stieltjes sense; $\mu(e)$ is a complex-valued additive function of Borel sets e; $J = J' + iJ''$ is the variable point with respect to which integration is performed; finally, $a(z)$ is a function, not given beforehand, analytic in K and depending on f. Conditions imposed on $f(z)$ in [2] were such that $\mu(e)$ could be determined as an absolutely continuous additive function of sets; accordingly (1.1) could be rewritten in the form

$$f(z) = \iint_K \frac{\varphi(J)dm(e_J)}{J - z} - a(z), \tag{1.2}$$

where $\varphi(J)$ is a complex valued function integrable over K (that is, the real and imaginary parts of $\varphi(J)$ are such) and $m(e)$ is the measure of e. In [2] we, thus, treat problems of the following type: *given a function of a given class, to solve the integral equation*

$$\iint_K \frac{\varphi(J)dm(e_J)}{J - z} - f(z) = a(z) \tag{1.3}$$

for the unknown $\varphi(J)$, where $\varphi(J)$ is to be of a certain class, while $a(z)$ is not given beforehand, but is to be analytic.

Our present purpose is to study problems of the following type, generalizing the problem (1.3).

Given a kernel (possibly complex valued) $k(z, J)$ and a function $f(z)$ of given classes, to solve the integral equation

$$\iint_K \frac{k(z, J)\varphi(J)dm(e_J)}{J - z} - f(z) = a(z) \tag{1.4}$$

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