UNIQUENESS OF THE INVERSE OF A TRANSFORMATION

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1. **Introduction.** In an earlier paper [4] the author has characterized both irreducibility and strong irreducibility of a continuous transformation $f(x)$ acting on a metric continuum in terms of the density of points whose images have unique inverses. These results were obtained in a simple way largely through the use of upper semi-continuity of certain real valued "diameter" functions associated with the given transformation. Recently G. S. Young [5] has shown that if the simple links [2] of a continuum are disjoint, uncountably many must be degenerate and, in a footnote, this result is formulated in the convenient language of continuous transformations. As such, of course, it takes the form of a theorem asserting that the inverse will be single valued for uncountably many image points under a particular kind of monotone mapping of a continuum onto a dendrite.

In this paper it will be shown first how the semi-continuity of the diameter function: $\phi(y) = \delta[f^{-1}(y)]$, $y \in B$, where $f(A) = B$ is the given mapping, can be made to yield in a very simple way much less restrictive conditions under which the points with a unique inverse will be uncountably everywhere dense in B. Other methods are then employed to show that for interior non-alternating transformations $f(M) = N$, where M is a continuum, every A-set, cut point, simple link, and end point of M is necessarily an inverse set.

It will be recalled that a continuous transformation $f(A) = B$ is (i) *monotone* if $f^{-1}(y)$ is a continuum for each $y \in B$, (ii) *non-alternating* if for $x, y \in B$, $f^{-1}(x)$ separates no two points of $f^{-1}(y)$ in A, and (iii) *interior* (or open) if the image of every open set in A is open in B. A subset X of A satisfying

$$X = f^{-1}f(X)$$

is called an *inverse set.*

In all our results it is understood that the spaces involved are separable and metric. For terminology and other definitions the reader is referred to the author's book *Analytic Topology* [3]. In particular it should be noted that a continuum is a compact connected set and that an A-set A in such a continuum M is a closed subset of M such that $M - A$ is the sum of a null sequence (G_i) of disjoint open sets each bounded by a single point of A. Also two points of such a continuum M are *conjugate* if no point separates them in M.

2. Uniqueness by means of cut points.

(2.1) Theorem. *If A is a continuum and $f(A) = B$ is continuous and such that the image points of cut points of A are dense in B and for each $y \in B$ any two*

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