REGULAR TRANSFORMATIONS

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1. **Introduction.** The notion of regular convergence as introduced by G. T. Whyburn [9], has been used by A. D. Wallace [7] to define a special kind of interior transformation, which he called 0-regular. The n-th dimensional generalization of these transformations was considered by W. T. Puckett [4]. In the following work almost all of the general theorems of Wallace's paper will be proved for the generalized transformations. It is also shown that the properties of being a lc^n (locally connected compactum for all dimensions $\leq n$) is invariant under an $(n - 1)$-regular transformation, and that this property is invariant under the inverse of an n-regular transformation.

In the entire paper it will be assumed that the sets on which our transformations act are compacta, and this hypothesis will be omitted from all the theorems. All of the ordinary complexes and cycles used shall have modulo two coefficients and the Vietoris cycles [5] used shall consist of these as coördinate cycles. Hereafter we shall refer to them simply as V-cycles.

2. **General Theorems.**

Definition. *The sequence of closed sets* (A_i), *contained in a compactum, are said to converge n-regularly to* A *if* $\lim A_i = A$, *and if corresponding to every* $\epsilon > 0$ *there is a* $\delta > 0$ *and an* N *such that if* V^r *is an r-dimensional V-cycle of* A_i *for* $i > N$ *and* $r \leq n$, *then* $V^r \sim 0$ *in a subset of* A_i *with diameter* $< \epsilon$.

This definition differs slightly from that of G. T. Whyburn's [9]. This would be r-regular convergence for $r \leq n$ under the original definition.

Definition. *The continuous single-valued transformation* $T(K) = K'$ *(where* K *will always be a compactum) will be called n-regular, provided that for every sequence of points* (y_i) *converging to* y *in* K', *we have that* $(T^{-1}(y_i))$ *converges n-regularly to* $T^{-1}(y)$ *in* K.

It is clear from the Eilenberg characterization of interior transformations [3; 174], that an n-regular transformation is interior. The following theorem, which will presently be useful, also follows readily from that characterization.

Theorem 2.1. *A necessary and sufficient condition that* $T(K) = K'$ *be interior is that for any* $e > 0$ *there exists a* $d > 0$ *such that if* $\rho(x, y) < d$ *in* K', *then* $\rho(T^{-1}(x), T^{-1}(y)) < e$ *in* K, *where* $\rho(T^{-1}(x), T^{-1}(y))$ *denotes Hausdorff distance.*

The next theorem follows in an entirely analogous manner by using a metric established by the author in the hyperspace of lc^n subsets of K [8].

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