ABEL TRANSFORMS OF TAUBERIAN SERIES

By Ralph Palmer Agnew

1. **Introduction.** Let $u_0 + u_1 + \cdots$ be a series of complex terms satisfying the Tauberian condition

$$n \mid u_n \mid < K \qquad (n = 0, 1, 2, \cdots). \tag{1.1}$$

Let $\sigma(t)$ denote the Abel transform of Σu_n :

$$\sigma(t) = \sum_{k=0}^{\infty} t^k u_k \qquad (0 < t < 1). \tag{1.2}$$

Let L denote the set of limit points of the sequence $s_0, s_1, \cdots$ of partial sums of Σu_n. Let L_A denote the set of limit points of the Abel transform $\sigma(t)$; $z'' \varepsilon L_A$ if there is a sequence $t_1, t_2, \cdots$ such that $0 < t_n < 1$, $t_n \to 1$, and $\sigma(t_n) \to z''$ as $n \to \infty$. If $\sigma(t) \to \sigma$ as $t \to 1$, then Σu_n is summable to σ by Abel's method A; but it is not assumed that Σu_n is summable A.

Hadwiger [1] proved that each of the following assertions is true when

$$\rho = 1.0160 \cdots \tag{1.3}$$

and false when

$$\rho < .4858 \cdots . \tag{1.4}$$

Assertion 1.1. *If Σu_n is a series satisfying the Tauberian condition $n \mid u_n \mid < K$, then to each $z' \varepsilon L$ corresponds a $z'' \varepsilon L_A$ such that*

$$\mid z' - z'' \mid \leq \rho \limsup_{n \to \infty} n \mid u_n \mid. \tag{1.5}$$

Assertion 1.2. *If Σu_n is a series satisfying the Tauberian condition $n \mid u_n \mid < K$, then to each $z'' \varepsilon L_A$ corresponds a $z' \varepsilon L$ such that* (1.5) *holds.*

As Hadwiger pointed out, his result implies that $L = L_A$ when Σu_n is a series satisfying the Tauberian condition $nu_n \to 0$. A Tauberian theorem of Littlewood [2] states that if Σu_n satisfies the weaker Tauberian condition (1.1) and Σu_n is summable A to σ, then Σu_n converges to σ. This implies that if Σu_n satisfies (1.1) and L_A contains exactly one point, then $L = L_A$. Hadwiger's result shows that (1.1) does not imply universal identity of L and L_A ; for otherwise the assertions would be true when $\rho = 0$.

It is the object of this note to prove the following theorem which gives the least constant ρ for which Assertion 1.1 is true.

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