FUNCTIONS OF EXPONENTIAL TYPE, IV

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1. In [1; 20] it was observed that an upper bound for the Whittaker constant is furnished by the absolute value r_n of the root of smallest absolute value of an exponential polynomial $f_n(z)$ satisfying $f_n'(z) = af_n(\epsilon z)$, where $|a| = 1$ and ϵ is a primitive n-th root of unity. I stated without proof that $r_5 > .84$ and that $\overline{\lim}\, r_n = \infty$ as $n \to \infty$. The first of these statements is incorrect, and the second is probably incorrect also; I had mistakenly supposed that it is indifferent which particular n-th root is chosen for ϵ. I have now computed r_5 for $f_5(z)$ when $\epsilon = e^{4\pi i/5}$, and find that

$$.7398 < r_5 < .7399.$$

Since Levinson [2] has given an improved lower bound for the Whittaker constant W, we can now state that

$$.728 < W < .7399.$$

The form of $f_5(z)$ most convenient for computation turns out to be

$$f_5(z) = \sum_{n=0}^{\infty} \frac{z^n e^{\frac{1}{2}n(n-1)}}{n!};$$

the value of r_5 is approximated by the root of smaller absolute value of $1 + A_5 z^5 + A_{10} z^{10} = 0$, where A_k is the coefficient of z^k in

$$f_5(z) f_5(\epsilon z) f_5(\epsilon^2 z) f_5(\epsilon^3 z) f_5(\epsilon^4 z).$$

More explicitly,

$$A_5 = (25 - 65\epsilon + 30\epsilon^2 + 20\epsilon^3 - 10\epsilon^4)/24,$$

$$A_{10} = (78125 - 515000\epsilon + 390000\epsilon^2 + 390000\epsilon^3 - 343125\epsilon^4)/(10!).$$

2. The conjecture that $W = 2/e$ was attributed to the wrong author by an error in the reference system of [1]: on page 17, line 11 of §1, read [6] instead of [5].

References

1. R. P. Boas, Jr., *Functions of exponential type*, II, this Journal, vol. 11(1944), pp. 17–22.
2. N. Levinson, *The Gontcharoff polynomials*, this Journal, vol. 11(1944), pp. 729–733.

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