FRÉCHET POLYHEDRA

By Harry D. Huskey

0. Introduction

0.1. Let us consider a continuous surface S in xyz-space. The Lebesgue area of such a surface is defined as follows:

$$A(S) = \inf (\liminf_n E(P_n)),$$

where the infimum of the limit inferiors is taken with respect to all sequences $\{P_n\}$ of polyhedra for which the distance between the surface S and the polyhedra converges to zero. $E(P_n)$ denotes the elementary area of the polyhedron P_n.

0.2. To make the above definition meaningful several concepts must be clarified. For example: What is a surface? What is the distance between two surfaces? What is meant by convergence? What is a polyhedron? What is its elementary area?

Several of these questions have been extensively studied in the literature [1], [3], [4]. It is the purpose of this paper to consider the last two of these questions.

0.3. There are several definitions of polyhedra in common use in the literature. Four such definitions are given here (cf. §2.2) and their equivalence is discussed (cf. §§4.1, 4.8). In this paper it is shown that for the purpose of defining the area of a surface these definitions are entirely equivalent (cf. §§4.5, 4.7, 4.9). This result is significant since there are situations where each definition has its advantage.

A polyhedron P is given by a region R and a triple of functions $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, with certain special properties, where (u, v) is in R. The region and the triple of functions will be called a representation of P (cf. §1.1). Now, a particular polyhedron will have many representations and since the area is defined in terms of a representation of a particular type (cf. §§2.5, 2.7) a pertinent question arises: Is the area independent of the representation? As shown indirectly in the literature [4; 352], the answer to this question is in the affirmative. A direct proof of this fact is contained in this paper (cf. §4.4).

0.4. This paper is divided into four parts as follows: Chapter one, entitled preliminaries, sketches a very brief introduction to the study of surfaces in parametric form; for a more extensive discussion see [1], [4]. Chapter two consists of definitions of polyhedra, area of a polyhedron, and areas of surfaces. In

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