ASYMPTOTIC SOLUTION OF BOUNDARY VALUE PROBLEMS FOR THE DIFFERENTIAL EQUATION $\Delta U + \lambda \frac{\partial}{\partial x} U = \lambda f(x, y)$

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1. This paper deals with the asymptotic solution of boundary value problems for the differential equation

$$\Delta U + \lambda \frac{\partial U}{\partial x} = \lambda f(x, y) \tag{1}$$

in two independent variables for large values of the real parameter λ. Let $U(x, y; \lambda)$ be the solution of (1) assuming prescribed values on the boundary of a domain. It will be proved that

$$\lim_{\lambda \to \infty} U(x, y; \lambda)$$

exists and satisfies the "limiting" differential equation $\partial U/\partial x = f(x, y)$. But the limit function, being a solution of a *first* order differential equation, will in general not assume the prescribed boundary values along the *whole* boundary. The parts of the boundary where the boundary condition ceases to be satisfied in the limit will be seen to depend on the sign of λ. At these boundary arcs the convergence of U cannot be uniform, since the limit function is there discontinuous.

The subject of this paper is a special case of a type of problem frequently occurring in mathematical physics. Whenever in a differential equation the coefficients of the terms of highest order are small by comparison with the other terms, the solution of a boundary problem for this differential equation can be expected to show irregularities owing to non-uniform dependence on the coefficients of the differential equation. The boundary layers of the flow of a fluid with low viscosity offer the best known example. Similar phenomena are met in the theory of elasticity, e.g., in the study of the buckling of plates [2]. Many other problems might be formulated as "boundary layer" phenomena.

In the boundary layer theory of hydrodynamics the convergence of the solution to a solution of the limiting differential equation is taken for granted for physical reasons. It is, however, easy to give examples of mathematically similar problems where the solution diverges. A purely mathematical approach to such questions is therefore desirable wherever the complexity of the problem does not preclude such a treatment.

2. Let B be a finite plane domain of finite connectivity bounded by a finite number of arcs with continuously turning tangent. The boundary of B will be

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