PROJECTIVE INVARIANTS OF A PAIR OF SURFACES

By Chuan-Chih Hsiung

1. **Introduction.** Let σ, σ^* be two elements of the second order of two surfaces at two ordinary points A, A^* in ordinary space. Buzano [2] has shown the existence of a projective invariant of σ, σ^* together with a metric characterization, provided the tangent planes of σ, σ^* at the points A, A^* are distinct. As a supplement to the investigation of Buzano, Bompiani [1] has obtained some projective characterizations of this invariant.

The purpose of the present paper is to study the other case, where the tangent planes of σ, σ^* at the points A, A^* are coincident.

We have demonstrated that for two plane curves having a common tangent at two ordinary points no projective invariant can be determined by the neighborhood of the second order of the two curves at these points [3]. But the corresponding result for a pair of surfaces is quite different, since we have here one projective invariant.

2. **A projective invariant.** Let S, S^* be two surfaces in ordinary space having a common tangent plane at two ordinary points A, A^*; and t, t^* the harmonic conjugate lines of AA^* respectively with respect to the asymptotic tangents of the surfaces S, S^* at the points A, A^*. Let the homogeneous projective coördinates of a point in the space be denoted by (x_1, x_2, x_3, x_4). If the points A, A^* and the intersection P of the tangents t, t^* be chosen respectively for the vertices (0, 0, 0, 1), (0, 1, 0, 0), (1, 0, 0, 0) of the tetrahedron of reference, then the power series expansions of the two surfaces in the neighborhood of the points A, A^* may be written in the form:

$$S: \quad \frac{x_3}{x_4} = l_{11}\left(\frac{x_1}{x_4}\right)^2 + l_{22}\left(\frac{x_2}{x_4}\right)^2 + \cdots, \tag{1}$$

$$S^*: \quad \frac{x_3}{x_2} = l_{33}\left(\frac{x_1}{x_2}\right)^2 + l_{44}\left(\frac{x_4}{x_2}\right)^2 + \cdots. \tag{2}$$

Let us now consider the most general projective transformation

$$\begin{aligned} x_1' &= a_{11}x_1 + a_{13}x_3, \qquad x_2' = a_{22}x_2 + a_{23}x_3, \\ x_3' &= a_{33}x_3, \qquad x_4' = a_{43}x_3 + a_{44}x_4, \end{aligned} \tag{3}$$

which leaves the points A, A^*, P invariant. The effect of this transformation on equations (1), (2) is to produce two other equations of the same form whose coefficients, indicated by accents, are given by the formulas:

Received January 15, 1943.