THE PROJECTIVE DIFFERENTIAL GEOMETRY OF A NON-HOLONOMIC HYPERSURFACE

By Buchin Su

In a previous paper [7] we have generalized the Moutard quadrics of a surface in ordinary space to a non-holonomic surface V_3^2 by showing that the locus of the osculating conics of sections of V_3^2 produced by planes through a non-asymptotic tangent of V_3^2 at a generic point O is a quadric. It is natural to inquire whether a similar result can be obtained for a non-holonomic hypersurface V_{n+1}^n in projective space S_{n+1} of $n + 1$ dimensions. We may propose a problem of finding an analogue of Čech's theorem concerning the sections of a holonomic hypersurface V_n produced by linear spaces $[\nu]$ of ν dimensions through a given space $[\nu - 1]$ in the tangent hyperplane of V_n at a generic point O, see [4; 618]. In this note we merely consider the problem in the cases $\nu = 2$ and $\nu = 3$; the other cases $\nu = 4, 5, \cdots$ cannot be decided so long as the notion of the Čech quadric for a hypersurface is not given in advance.

1. Suppose that a non-holonomic hypersurface V_{n+1}^n in S_{n+1} is defined by an equation of Pfaff, namely,

$$dz = p_{(\sigma)} dx^\sigma, \tag{1}$$

where the functions $p_{(\sigma)} = p_{(\sigma)}(x^1, x^2, \cdots, x^n, z)$ do not satisfy the conditions of integrability, and use is made of the convention that when the same index appears as a subscript and a superscript in a term this term stands for the sum of the terms obtained by giving the index each of its n values $1, 2, \cdots, n$. At a generic point O of V_{n+1}^n we can determine a projective system of reference as follows:

The hyperplane which corresponds to a point $P(x^1, x^2, \cdots, x^n, z)$ is given by the equation

$$Z - z = p_{(\sigma)}(X^\sigma - x^\sigma). \tag{2}$$

Especially if the tangent hyperplane of V_{n+1}^n at O is taken for the coördinate hyperplane $Z = 0$, then *at the origin* $O(0, 0, \cdots, 0)$ we have

$$p_{(1)} = p_{(2)} = \cdots = p_{(n)} = 0. \tag{3}$$

The intersection of the hyperplane $Z = 0$ and the corresponding hyperplane at the consecutive point $O'(dx^1, dx^2, \cdots, dx^n, dz)$ of O gives a correspondence between a line through O and a space $[n - 1]$ in $Z = 0$, namely, the projectivity of the cell of Bompiani [1] for the line

Received June 8, 1942. The author is Research Professor in Mathematics in the Academia Sinica.