SYSTEMS OF QUADRICS ASSOCIATED WITH A POINT OF A SURFACE, II

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1. **Introduction.** In [4], the author defined a pair of quadrics called the *chord section quadrics*. It will be of interest to consider the plane sections of the asymptotic ruled surfaces $R_\lambda^{(u)}$ and $R_\lambda^{(v)}$ generated respectively by the asymptotic u- and *v-tangent* along a curve C_λ on a non-ruled analytic surface σ. The purpose of this note is to study several projectively covariant figures of the surface σ by means of the Moutard quadrics $Q_n^{(u)}$, $Q_n^{(v)}$ of $R_\lambda^{(u)}$, $R_\lambda^{(v)}$, namely, *the asymptotic section quadrics* belonging to a non-asymptotic tangent t_n. In particular, a simple definition of the projective normal is given; the relations between these quadrics and the pangeodesics are studied in detail; the union curves of a certain canonical line congruence are introduced and, finally, many new canonical rays are interpreted.

2. **Analytic basis.** We need to recall some formulas utilized in the subsequent discussion. Let σ be a non-ruled non-degenerate surface in ordinary space; then the normal coördinates x of Fubini of a generic point on σ are solutions of the following differential equations:

$$
\begin{aligned}
x_{uu} &= px + \theta_u x_u + \beta x_v \\
x_{vv} &= qx + \gamma x_u + \theta_v x_v
\end{aligned}
\qquad (\theta = \log \beta\gamma), \tag{1}
$$

where the parametric curves u, v are asymptotic curves of σ. The formulas for the third and the fourth derivatives of x expressed in terms of x, x_u, x_v, x_{uv} are as follows:

$$
\begin{aligned}
x_{uuu} &= (p_u + p\theta_u)x + (p + \theta_u^2 + \theta_{uu})x_u + (\beta_u + \beta\theta_u)x_v + \beta x_{uv}, \\
x_{uuv} &= (p_v + \beta q)x + (\theta_{uv} + \beta\gamma)x_u + \pi x_v + \theta_u x_{uv}, \\
x_{uvv} &= (q_u + \gamma p)x + \chi x_u + (\theta_{uv} + \beta\gamma)x_v + \theta_v x_{uv}, \\
x_{vvv} &= (q_v + q\theta_v)x + (\gamma_v + \gamma\theta_v)x_u + (q + \theta_v^2 + \theta_{vv})x_v + \gamma x_{uv};
\end{aligned} \tag{2}
$$

and

$$
\begin{aligned}
x_{uuuv} &= [p(\theta_{uv} + \beta\gamma) + \theta_u(\beta q + p_v) + (\beta q)_u + p_{uv}]x + (*)x_u \\
&\quad + (*)x_v + [\theta_{uu} + \theta_u^2 + \pi]x_{uv}, \\
x_{uuvv} &= [q_{uu} + (\gamma p)_u + p\chi + \theta_v(p_v + \beta q)]x + (*)x_u + (*)x_v \\
&\quad + [2\theta_{uv} + \theta_u\theta_v + \beta\gamma]x_{uv},
\end{aligned} \tag{3}
$$

Received April 15, 1942; in revised form, March 15, 1943.