SYSTEMS OF QUADRICS ASSOCIATED WITH A POINT OF A SURFACE, I

By George Wu

1. **Introduction.** The projective theory of a surface in ordinary space has been enriched, in most part, by means of certain covariant quadrics projectively associated with a point of the surface. Especially the Moutard quadrics [8] play an important rôle, because each of them depends not only a point of the surface, but also on the direction of a tangent.

On a non-ruled non-degenerate surface S in ordinary space S_3, let us select a curve C and two points x and $P \equiv P_0$ on C. Denote the other points of intersection of the two asymptotic curves through each of these points by P_1 and P_2. As P varies on the curve C, the asymptotic chord PP_1 generates a ruled surface R_1. For an arbitrary tangent t_n of S at x we have the Moutard quadric $Q^{(1)}$ of R_1 and, similarly, the Moutard quadric $Q^{(2)}$ of R_2 generated by the chord PP_2. These two quadrics $Q^{(1)}$ and $Q^{(2)}$ are called the *chord section quadrics*, since they are analogues of the asymptotic chord quadrics of Bompiani [5].

The object of this paper is to give a new construction of the Darboux pencil, a new geometrical interpretation of the Moutard correspondence, and some properties of the chord section quadrics.

2. **Analytic basis.** If we take the asymptotic parameters u, v as the independent variables of a point x on S, then the four projective homogeneous coördinates of x are independent solutions of a completely integrable system of differential equations in Fubini's canonical form:

$$(1) \qquad \begin{aligned} x_{uu} &= \theta_u x_u + \beta x_v + px \\ x_{vv} &= \gamma x_u + \theta_v x_v + qx \end{aligned} \qquad (\theta = \log \beta\gamma),$$

where the coefficients are functions of u and v, and satisfy certain integrability conditions.

From (1) we can calculate the third and fourth derivatives of x in terms of x, x_u, x_v, x_{uv}, namely,

$$(2) \qquad \begin{aligned} x_{uuu} &= (*)x + (\theta_{uu} + \theta_u^2 + p)x_u + (\beta_u + \beta\theta_u)x_v + \beta x_{uv}, \\ x_{uuv} &= (*)x + (\theta_{uv} + \beta\gamma)x_u + \pi x_v + \theta_u x_{uv}, \\ x_{uvv} &= (*)x + \chi x_u + (\theta_{uv} + \beta\gamma)x_v + \theta_v x_{uv}, \\ x_{vvv} &= (*)x + (\gamma_v + \gamma\theta_v)x_u + (\theta_{vv} + \theta_v^2 + q)x_v + \gamma x_{uv}; \end{aligned}$$

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