LIMITS TO THE CHARACTERISTIC ROOTS OF A MATRIX

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1. **Introduction.** If A is a square matrix of order n and I is the unit matrix, the *characteristic equation* of A is defined to be the equation obtained by equating to zero the determinant $|\lambda I - A|$. The roots of this equation are called the *characteristic roots* of A.

Several authors (for a list of references, see [1]) have given upper limits to the characteristic roots. In one of the more recent of these papers this author [2] proved the

THEOREM. *If $A = (a_{ij})$ is any square matrix, and if*

$$2S_r = \sum_{j=1}^{n} \{|a_{rj}| + |a_{jr}|\}, \qquad 2S'_r = \sum_{j=1}^{n} |a_{rj} + a^*_{jr}|,$$

$$2S''_r = \sum_{j=1}^{n} |a_{rj} - a^*_{jr}|,$$

where a^* denotes the conjugate imaginary of a, and if S, S', S'' are the greatest of the S_r, S'_r, S''_r, respectively, then for any characteristic root $\lambda = \alpha + i\beta$ of A it is true that

$$|\lambda| \leq S, \qquad |\alpha| \leq S', \qquad |\beta| \leq S''.$$

In the present note both upper and lower limits to the characteristic roots and to their real and imaginary parts are given. These limits are not always better than those previously given. However, these limits are usually better for matrices in which the absolute values of the diagonal elements are large as compared with the absolute values of the other elements; and they are better for matrices with elements of one row or one column large in absolute value as compared with other elements.

2. **Definitions of symbols used.** It is convenient to list here the symbols which will be used in the discussion to follow. These are defined for the matrix $A = (a_{ij})$ of order n as follows:

$$R_i = \sum_{j=1}^{n}{}' |a_{ij}|, \qquad R'_i = \sum_{j=1}^{n}{}' |a_{ji}|,$$

where

$$\sum_{j=1}^{n}{}' a_{ij} = \sum_{j=1}^{n} a_{ij} - a_{ii}, \qquad \sum_{j=1}^{n}{}' a_{ji} = \sum_{j=1}^{n} a_{ji} - a_{ii},$$

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