A PROBLEM OF SET-THEORETIC TOPOLOGY

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Introduction

The present paper is concerned with the problem of determining under what conditions a topological space can be resolved into two complementary sets each of which is dense in the given space. A space admitting such a resolution is said to be *resolvable*; a space (satisfying a certain restriction to be specified below) which cannot be so resolved is said to be *irresolvable*. Several of the techniques made use of in obtaining a partial answer to this question have applications to other topological problems. We shall indicate such applications where they occur.

The principal construction, indeed, utilized in producing irresolvable topological spaces is a special case of an operation to which any topological space may be subjected and which we have named *topological expansion*. The first chapter is accordingly devoted to the development of a theory of such expansions. We investigate properties of spaces which are preserved under arbitrary expansions, giving in this connection a separation axiom of peculiar interest first stated by Urysohn. In considering various types of expansions, we prove the existence of a particular expansion enjoying a number of curious properties. This expansion, called a *maximal expansion*, is essential in the construction of irresolvable spaces. We also consider various properties of contraction, the operation inverse to expansion.

The second chapter contains a study of irresolvable spaces. The existence of irresolvable connected T_1-spaces, totally disconnected Urysohn spaces, and totally disconnected completely regular spaces is proved directly from the expansion theory. Some irresolvable spaces enjoy a very strong disconnectivity property, which we investigate in detail.

In the final chapter, it is shown that metric spaces, bicompact Hausdorff spaces, spaces satisfying the first countability axiom of Hausdorff, and various other special spaces are all resolvable. We also obtain a reduction showing that only T_1-spaces, and in fact only bicompact T_1-spaces, need be considered in dealing with the resolution problem.

A brief statement of the provenience of the problem may be appropriate. In studying inclusion relations possible among the sets formed from a given subset A of a space R by iteration of complements and closures [4], the writer

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