COVERING MAPPINGS

By Alfred M. Peiser

In this paper, the covering spaces C of a topological space K are defined by means of the subgroups of the fundamental group F of K associated with them. A covering mapping of C into itself is a mapping in which each point P of C goes into a point P' such that P and P' are "over" the same point of K. These mappings are similar to but more general than Deck transformations which are restricted to the so-called "regular" covering spaces [2; 195–196]. Every covering mapping is shown to be generated by an element of F, and given a C, conditions are found on an element of F which determine whether or not it will generate a covering mapping or homeomorphism of C. These conditions seem to be tacitly assumed in most standard works. See, for example, [2; 198].

We take K to be connected, locally connected and locally simply connected. "Paths" of K are continuous maps of the unit interval, $0 \leq t \leq 1$ on K, called $a(t)$, etc., with end points $a(0)$ and $a(1)$. We assume the usual rules of operation for these paths [1; 217–219], and we shall write $ab(t)$ for the product $a(t)b(t)$. A fixed point P is taken as $a(0) = a(1)$ for the paths which generate F.

If G is a subgroup of F, then for each $a(t)$ with $a(0) = P$, $R(a)$ will denote the class of all $b(t)$ such that $ab^{-1}(t) \subset G$, where $\subset$ means "generates an element of". We denote by $C(G)$, or simply C, the set of all classes $R(a)$, and define a neighborhood $U(a)$ of $R(a)$ as the set of all classes $R(ax)$, where $x(t)$ is any path of K with $x(0) = a(1)$, and contained in a given neighborhood U of $a(1)$. Then C is a topological space, and is called a covering space of K. Let

$$S[R(a)] = a(1). \tag{1}$$

Then it can be shown that S is bi-continuous and locally homeomorphic. The proof is similar to that given in [1; 222].

Definition. A continuous mapping N of C onto a subset of itself is called a *covering mapping* if, for each point $R(a)$ of C,

$$S[NR(a)] = a(1). \tag{2}$$

A covering mapping which is a homeomorphism is called a *covering homeomorphism*.

For $Q \in F$, let $q(t)$ be a closed path beginning at P which defines Q. We denote by N_Q any fixed correspondence such that for each point $R(a)$ of C,

$$N_Q R(a) = R(qa). \tag{3}$$

Theorem 1. *If $QXQ^{-1} \in G$ for all $X \in G$, then N_Q is a covering mapping. If, in addition, $Q^{-1}XQ \in G$ for all $X \in G$, then N_Q is a covering homeomorphism.*

Received September 14, 1942. The author wishes to thank Professor W. W. Flexner for his aid in the preparation of this paper.