POWER SUMS OF POLYNOMIALS IN A GALOIS FIELD

By Herbert Leonard Lee

1. **Introduction.** Let $GF(p^n)$ denote the Galois field [4; 1–70] of order p^n. Let

$$M = c_0 x^m + c_1 x^{m-1} + \cdots + c_{m-1} x + c_m$$

denote a polynomial in an indeterminate x, with coefficients in $GF(p^n)$. If $c_0 \neq 0$, we write $\deg M = m$, and if $c_0 = 1$, we call M *primary*. In this paper we evaluate certain power sums, viz.

$$S_m^k = \sum_{\substack{\deg M = m \\ M \text{ primary}}}' M^k, \qquad R_m^k = \sum_{\substack{\deg M < m \\ \text{all } M \neq 0}} M^k,$$

$$\sigma_m^k = \sum_{\substack{\deg M = m \\ M \text{ primary}}}' \frac{1}{M^k}, \qquad \rho_m^k = \sum_{\substack{\deg M < m \\ \text{all } M \neq 0}} \frac{1}{M^k}.$$

In §3 we discuss some of the conditions under which the power sums vanish. §§4–6 are devoted to the evaluation of certain power sums in which the exponent k is of a special form. In §7 we evaluate R_m^k and S_m^k, where $k = a_m p^{nm} + a_{m-1} p^{n(m-1)} + \cdots + a_1 p^n + a_0$, for in this case R_m^k and S_m^k are given in one term only. §8 gives the development of recursion formulas involving power sums. Finally, in §9, we show the connection between R_m^k and ρ_m^k and the complete symmetric polynomial.

2. **Definitions and notation.** Let [1; 141–143]

$$\psi_m(t) = \prod_{\deg M < m} (t - M), \qquad \psi_0(t) = t, \tag{2.1}$$

where t is another indeterminate, and the product extends over all polynomials (including 0) of degree less than m; we then have

$$\psi_m(t) = \sum_{j=0}^{m} (-1)^{m-j} \begin{bmatrix} m \\ j \end{bmatrix} t^{p^{nj}}, \tag{2.2}$$

where

$$\begin{bmatrix} m \\ j \end{bmatrix} = \frac{F_m}{F_j L_{m-j}^{p^{nj}}}, \qquad \begin{bmatrix} m \\ 0 \end{bmatrix} = \frac{F_m}{L_m}, \qquad \begin{bmatrix} m \\ m \end{bmatrix} = 1, \tag{2.3}$$

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