MAPPINGS OF 2-MANIFOLDS INTO A SPACE

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Introduction

Given a complex K and a space T, let us consider all continuous mappings of K into T. These mappings are divided into homotopy classes, the mappings in any class being homotopic to each other. It is an important problem in topology to determine all these classes. However, the problem is completely solved only for some particular cases, notably for the classification of mappings of an n-complex into an n-sphere as given by the theorem of Hopf-Whitney. (See [2], [4], [5], [7], [17]. For a study of the mappings of one surface into another, see [6]. See also [12], [16].) For most cases, the space T is subject to various restrictions, such as being aspherical, or locally contractable. For the definitions, see [8], [9], [10], [11].

In a recent paper [14], Robbins studied the mappings of a 2-complex into a perfectly general space T. He obtained a necessary and sufficient condition for the homotopy of two "normal" mappings. But the condition is so complicated that it seems to be difficult to apply it to concrete cases. In the present work we shall take K as a 2-manifold, and obtain conditions for homotopy which are easier to apply. (Throughout this paper, we exclude the 2-sphere from 2-manifolds.) We shall carry out the classifications for some particular spaces T. The methods used both in Robbins' paper and the present work are due essentially to Hassler Whitney, who, in an unpublished paper, has classified mappings into projective spaces.

Homotopy groups were introduced by W. Hurewicz as a generalization of the fundamental group. See [3], [8], [9], [11], and, in particular, [10]. Although our present knowledge of these groups is still very imperfect, they have proved to be a powerful instrument in the classification problem. In the classification of mappings, it is sufficient to consider a particular class of mappings which are simpler than the general ones but are perfectly general in the sense of homotopy. This leads to the concept of normal mapping. In I and II we shall define homotopy groups and normal mappings and give some of their fundamental properties.

In III we shall study the mappings of a 2-manifold M^2 into a space T. It is well known that homotopic mappings induce the same homomorphisms to within inner automorphisms of the fundamental group G' of M^2 into that of T. Corresponding to each such homomorphism, defined say by f, a subgroup $\pi_f^2(T)$ of the 2-dimensional homotopy group $\pi^2(T)$ is defined. This subgroup plays an important rôle in the classification of mappings of M^2 into T.

A pseudo-projective plane of order m is the space defined by the fundamental

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