THE SINGULARITY S_1^m OF A PLANE CURVE

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1. By a singularity S_1^m of a plane curve we mean the point at which the tangent to the curve has a contact of order m with the curve. If this point is taken for the origin $O(0, 0, 1)$ and the tangent for $y = 0$, then the expansions of the curve in the neighborhood of O become

$$(1) \qquad x = s \sum_0^\infty a_\nu s^\nu, \qquad y = s^m \sum_0^\infty b_\nu s^\nu, \qquad z = 1 + \sum_0^\infty c_\nu s^\nu, \qquad a_0 b_0 \neq 0.$$

In particular, when $m = 3$, the singularity which is a point of inflexion has been studied by E. Bompiani [1] and the author [2]. It is B. Su [4], [5] who generalizes Bompiani's osculants to a curve with a representable singularity of high order. In a recent paper [3] we have studied the singularity S_1^4 in detail and obtain the canonical expansions of two species of S_1^4 that had been classified projectively.

It is natural to extend our method of representing the neighborhood of various orders of an S_1^4 to the study of an S_1^m $(m > 4)$. Here we investigate only the representable singularity considered by Su, namely, the singularity for which the invariant point O_{2m} exists, and give a geometrical interpretation of the conditions for a representable S_1^m, as these have been derived analytically by Su.

There are other covariant figures, besides O_{m+1}, l_{2m-1} and O_{2m}, determined by the neighborhoods of high orders of a representable S_1^m. A formulation of these elements as well as a supplement to the canonical expansion of Su for two species of a representable S_1^m forms the main object of this note.

2. Suppose that a curve C has a singular point S_1^m at $O(0, 0, 1)$, so that the expansion can be written in the form (1). In what follows we shall utilize an algebraic curve C_m of order m having a node, a singular point S_1^m, and an $(m - 2)$-ple point with coincident tangent. Let $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ be the $(m - 2)$-ple point and the node of C_m, respectively, and let

$$\frac{x_1 y - y_1 x}{y_1} + \rho \frac{\omega_1 x + \omega_2 y + \omega_3 z}{y_1 y_2} = 0$$

be the equation of the common tangent of C_m at P_1; the equation of C_m, which has a contact of order m with C at O, can be written as

$$y^2 \left(\frac{x_1 y - y_1 x}{y_1} + \rho \frac{\omega_1 x + \omega_2 y + \omega_3 z}{y_1 y_2} \right)^{m-2} - 2 \frac{y_1 y_2}{\omega_3} y \left(\frac{x_1 y - y_1 x}{y_1} \right)^{m-1}$$

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