THE POINT OF INFLEXION OF A PLANE CURVE

By Su-Cheng Chang

1. In attempting to generalize Bompiani's investigation for an inflexion of a plane curve [1] to the singularity of high order [3], the quartics of two cusps and an inflexion with concurrent cusp tangents and inflexion tangent are utilized here to represent several orders of the neighborhoods of the curve at its inflexion, as Bompiani has done by using cusped cubics. According as the inflexion is of the first or the second class, two types of canonical expansions are given, namely,

$$y = x^3 + \frac{27}{16} x^7 + qx^8 + (9),$$

$$y = x^3 + \frac{88}{3} x^8 + (9);$$

the former coincides with that given by Bompiani [1], while the latter is new. A similar improvement for the cusp of a plane curve has been established by the author [2].

2. There are quartics each of which has a point of inflexion at $O(0, 0, 1)$ and two cusps at $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$. In the following lines we utilize a special kind of them whose cusp tangents and inflexion tangent are concurrent. If the inflexion tangent is taken for $y = 0$, then the equation of one of these quartics is

$$\begin{aligned} &\left(\alpha \frac{x_1 y - y_1 x}{y_1} + \frac{\omega_1 x + \omega_2 y + \omega_3 z}{y_1 y_2}\right)^2 \\ &\quad \times \left(\frac{\alpha^2}{\tau^2} y^2 - 2\frac{\alpha^2}{\tau} \frac{x_1 y - y_1 x}{y_1} y - 2\frac{\alpha}{\tau} y \cdot \frac{\omega_1 x + \omega_2 y + \omega_3 z}{y_1 y_2}\right) \\ &\quad + 4\alpha^3 \left(\frac{x_1 y - y_1 x}{y_1}\right)^3 \left(\frac{\omega_1 x + \omega_2 y + \omega_3 z}{y_1 y_2}\right) \\ &\quad + 6\frac{\alpha}{\tau} y \left(\alpha \frac{x_1 y - y_1 x}{y_1} + \frac{\omega_1 x + \omega_2 y + \omega_3 z}{y_1 y_2}\right)\left(\frac{\omega_1 x + \omega_2 y + \omega_3 z}{y_1 y_2}\right)^2 \\ &\quad + \alpha^4 \left(\frac{x_1 y - y_1 x}{y_1}\right)^4 - 4\left(1 - \frac{\alpha^2 \tau^3}{a}\right)\frac{\alpha}{\tau} y \left(\frac{\omega_1 x + \omega_2 y + \omega_3 z}{y_1 y_2}\right)^3 = 0, \end{aligned} \tag{1}$$

Received April 22, 1942.