THE ABSOLUTE CONVERGENCE OF FOURIER SERIES

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1. **Introduction.** In this note we suppose throughout that the functions f_1, f_2 and f are integrable and periodic with period 2π. $f(x)$ is said to be a Young's continuous function [9], if there exist two functions f_1 and f_2 of the Lebesgue class $L^2(-\pi, \pi)$, satisfying

$$f(x) = \frac{1}{\pi} \int_{-\pi}^{\pi} f_1(\xi) f_2(\xi + x) \, d\xi.$$

The necessary and sufficient condition for the absolute convergence of a trigonometric series in the whole interval is that the series be a Fourier series of a Young's continuous function [3], [4]. One object of this paper is to obtain Young's functions with conditions imposing more on f_1 and less on f_2. Indeed, we shall prove

Theorem 1. *If* $f_1 \varepsilon$ Lip (α, p), $\alpha p > \frac{1}{2}$, $2 \geq p > 1$, $\alpha\alpha \leq 1$ *and* $f_2 \varepsilon$ Lip $(1/2p, q)$ *for* $q > 1$, *then the Fourier series*

$$f(x) = \frac{1}{\pi} \int_{-\pi}^{\pi} f_1(\xi) f_2(\xi + x) \, d\xi$$

converges absolutely in the whole interval.

The notation $\varphi \varepsilon$ Lip (α, p) means that

$$\left(\int_{-\pi}^{\pi} | \Delta\varphi |^p \, d\theta \right)^{1/p} = O(h^\alpha) \tag{1.1}$$

as $h \to +0$ and $\Delta\varphi$ denotes one of the three differences [5]

$$\varphi(\theta) - \varphi(\theta - h), \qquad \varphi(\theta + h) - \varphi(\theta), \qquad \varphi(\theta + h) - \varphi(\theta - h).$$

If $f(\theta) \varepsilon$ Lip (α, p) and

$$f(\theta) \sim \sum_{m=-\infty}^{\infty} c_m e^{im\theta}, \tag{1.2}$$

then the series

$$\sum_{m=-\infty}^{\infty} | c_m |^k \tag{1.3}$$

is convergent for $k > p/(p + \alpha p - 1)$, if

$$0 < \alpha \leq 1, \qquad 1 < p \leq 2.$$

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