ASYMPTOTIC DEVELOPMENTS OF CERTAIN INTEGRAL FUNCTIONS

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I. Theorems of Ford and Newsom

1. **Introduction.** In several papers extending over the years 1904-08, E. W. Barnes (see especially [1]) determined the asymptotic behavior in the neighborhood of the point at infinity of a number of analytic functions defined by their Maclaurin developments. Moreover, several investigations of a similar nature have been made very recently by E. M. Wright [5], [6], [7]. The work of Barnes and Wright, in each instance, consists largely of a detailed study of the particular function considered. On the other hand, W. B. Ford [2; 4–15, 30–37] and C. V. Newsom [4] have recently established certain theorems which are general in character and which may be applied to a variety of different functions. In fact, they may be used to obtain the asymptotic developments of several of the specific functions considered by Barnes and Wright.

The present paper presents an application of the theorems of Ford and Newsom. A certain extension of Newsom's theorem is first stated, the proof being omitted. We then proceed to determine the asymptotic developments of the general integral function

$$F_\alpha(z) = \sum_{n=0}^{\infty} \frac{h(n)z^n}{\Gamma(\alpha n + t)} \qquad (\alpha > 0), \tag{1.1}$$

where t is any constant, real or complex, and where the function $h(n)$ depends only on n and satisfies certain further conditions. This work constitutes Part II of the paper. In Part III, we apply the theorem obtained in Part II to the special function

$$E_\alpha(z, \theta, \beta) = \sum_{n=0}^{\infty} \frac{z^n}{(n + \theta)^\beta \, \Gamma(\alpha n + 1)} \qquad (\alpha > 0), \tag{1.2}$$

where θ and β are any constants, real or complex, except that θ cannot equal zero or a negative integer. The asymptotic developments of both the functions given by (1.1) and (1.2) have been discussed by Ford for the special case in which $\alpha = 1$. His results are, in fact, a special case of those obtained in Parts II and III of this paper.

The most recent work on the function (1.1) appears to be that of Wright [5]. We shall refer to it later. Barnes has investigated the function (1.2) under the condition that θ is not an integer.

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