THE RECIPROCAL OF CERTAIN TYPES OF HURWITZ SERIES

BY L. CARLITZ

1. Introduction. By (integral) Hurwitz series we shall mean series of the form

$$f(u) = \sum_{m=1}^{\infty} \frac{A_m}{g_m} u^m, \tag{1.1}$$

where $A_m = A_m(x)$ is a polynomial in an indeterminate x with coefficients in a fixed Galois field $GF(p^n)$, and the denominator is defined by

$$g_m = g(m) = F_1^{a_1} F_2^{a_2} \cdots F_s^{a_s} \qquad (g_0 = 1),$$

where

$$m = a_0 + a_1 p^n + \cdots + a_s p^{ns} \qquad (0 \leq a_i < p^n),$$

and

$$F_k = (x^{p^{nk}} - x)(x^{p^{nk}} - x^{p^n}) \cdots (x^{p^{nk}} - x^{p^{n(k-1)}}) \qquad (F_0 = 1).$$

For properties of Hurwitz series required here see [1; esp. 507-509]. We are interested in the coefficients of $u/f(u)$.

First, however, we consider the "linear" case

$$f(u) = \sum_{i=0}^{\infty} \frac{A_i}{F_i} u^{p^{ni}} \qquad (A_0 = 1), \tag{1.2}$$

where again A_i is integral, that is, a polynomial in x. Now the inverse of (1.2) has the same general form, say

$$\lambda(u) = \sum_{i=0}^{\infty} \frac{E_i}{F_i} u^{p^{ni}} \qquad (E_0 = 1), \tag{1.3}$$

with integral E_i. In a previous paper [2] we assumed

$$E_i \equiv 0 \qquad (\text{mod } g(p^{ni} - 1)). \tag{1.4}$$

We now make the milder assumption

$$E_i \equiv 0 \qquad (\text{mod } L_{i-1}) \tag{1.5}$$

for $i > 0$, where

$$L_k = (x^{p^{nk}} - x)(x^{p^{n(k-1)}} - x) \cdots (x^{p^n} - x) \qquad (L_0 = 1).$$

Define β_m by means of

$$\frac{u}{f(u)} = \sum_{m=0}^{\infty} \beta_m \frac{u^m}{g_m}$$

Received April 9, 1942.