A PARTICULAR SET OF TEN POINTS IN SPACE

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1. **Introduction.** A generic trilinear form $T(2, 3, 4) = (\alpha x)(\beta y)(\gamma z)$ with digredient variables x, y, z which are contragredient to ξ, η, ζ respectively in their spaces [2], [3], [4] depends upon 12 absolute constants. There are 10 pairs $x, y = p_i, q_i$ $(i = 1, \cdots, 10)$ which are neutral for z in $T = 0$. In an earlier paper [4], it is proved that the set of ten points p_i, P_{10}^2 and the set of ten points q_i, Q_{10}^3 are connected by the double identity in ξ, η,

$$(1) \qquad \sum_{i=1}^{i=10} (q_i \eta) \cdot (p_i \xi)^2 \equiv 0.$$

In this identity the set P_{10}^2 is generic with 12 absolute constants. The set Q_{10}^3 is then projectively determined by the identity and thus is subject to three projective conditions. It is the purpose of this paper to determine the nature of these conditions, and so explore some of their consequences.

2. **The generic character of 9 points of Q_{10}^3.** In this section we prove that the three conditions on Q_{10}^3 all fall on the tenth point when the first nine are given generically. This is somewhat unusual. For example, the ten nodes of a rational sextic are subject to three conditions and only eight can be chosen generically; in space the nine nodes of a symmetroid are subject to three conditions and only seven can be chosen generically. We observe first that the squares $(p\xi)^2$ of points p in [2] represent a mapping of the plane [2] upon the points r of a Veronese V_2^4 in [5], the point p_i mapping into a point r_i. Thus we have a set R_{10}^5 on V_2^4, and the identity (1) asserts that the set Q_{10}^3 is associated to the set R_{10}^5. Hence

(1) *The three conditions on Q_{10}^3 appear in its associated set R_{10}^5 as the three conditions that R_{10}^5 is on a Veronese V_2^4.*

For, it is known [**3**; Theorem 18] that on nine generic points in [5] there are four V_2^4's, whence the three conditions on R_{10}^5 fall on the tenth when the first nine are given. Let then $r_1, \cdots, r_9$ be projected from r_{10} into a set $S_9^4 = s_1, \cdots, s_9$ in [4], the V_2^4 on R_{10}^5 projecting into a M_2^3 on S_9^4. This S_9^4 is associated to $Q_9^3 = q_1, \cdots, q_9$. Again it is known [**3**; Theorem 15] that on an S_9^4 there are two M_2^3's, these being paired with the two reguli on the associated Q_9^3. Since any M_2^3 in [4] is the map of the plane by conics on a point, any S_9^4 and M_2^3 on it can be obtained by such a mapping from nine points of the plane. Since the above S_9^4 is obtained from $p_1, \cdots, p_9$ by the mapping with conics on p_{10}, the S_9^4 is a generic set, and its associated set Q_9^3 is also generic. Hence

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