# THE DOUBLE-$N_n$ CONFIGURATION

By Arthur B. Coble

1. **Introduction.** We are concerned initially only with the double-$N_n$ configuration defined by a White [7] surface, a configuration in the linear space $[n + 1]$ which consists of $N_n$ lines, $l_i$, and of $N_n$ spaces $[n - 1]$, $\lambda_j$, such that $l_i$ and $\lambda_j$ are incident if and only if $i \neq j$ $(i, j = 1, \cdots, N_n)$, where $N_n$ is the binomial coefficient $(n + 2; 2)$. In conclusion, however, we raise the question as to whether there may not well be configurations of this sort more general than those which define, and are defined by, a White surface.

The White surface, $W_n$, in $[n + 1]$ is the map of the plane by the linear system of curves of order $n + 1$ on a generic set, $P^2_{N_n}$, of points $p_1, \cdots, p_{N_n}$ of the plane. It has the order $(n + 1)^2 - N_n = N_{n-1}$. The directions about a point $p_i$, say $p_i^*$, map into points on a line $l_i$ of the configuration $CW_n$ under discussion. Let $C_j$ be the curve of order $n$ on all of the points of $P^2_{N_n}$ except $p_j$. Since $P^2_{N_n}$ is generic, the curves $C_j$ are all distinct and generic. A particular curve $C_j$ maps into a curve $k_j$ in an $[n - 1]$ of order $N_{n-2}$ which crosses each of the lines $l_i$ $(i \neq j)$, since $C_j$ goes through $p_i$ with some definite direction. Let $\lambda_j$ be the $[n - 1]$ in which $k_j$ lies. Then $\lambda_j$ also cuts $l_i$ if $i \neq j$. We thus obtain from $P^2_{N_n}$ the double-$N_n$ configuration of the type we will call $CW_n$. The configuration itself, apart from the $W_n$ which defines it, is formally self-dual. The lines and $[n - 1]$'s are dual in $[n + 1]$. The non-incidence of $l_i$ and $\lambda_i$ imply that they have neither a $[0]$ nor a $[n]$ in common. The incidence of $l_i$ and $\lambda_j$ $(j \neq i)$ imply that they have a point $m_{ij}$ in common and a prime $\mu_{ij}$ in common.

The first instance for $n = 1$, $N_n = 3$ is the figure of three lines $l_1, l_2, l_3$ in the plane $W_1$ and three points $\lambda_1, \lambda_2, \lambda_3$, each point on two of the lines, i.e., a plane triangle. The mapping mentioned above by conics on $P^2_3$ is a quadratic transformation from the plane of $P^2_3$ to $W_1$. In this case the $CW_1$ has no geometric interest.

The second instance for $n = 2$, $N_n = 6$ is the figure of six skew lines $l_1, \cdots, l_6$ on a cubic surface $W_2$ in [3] and the six lines $\lambda_1, \cdots, \lambda_6$, which with $l_1, \cdots, l_6$ form a "double-six" on the surface. This figure has two significant properties for which we use the terms *descriptively self-dual* and *intrinsically self-dual* with the following meanings. A formally self-dual configuration is descriptively self-dual if for every figure constructed from its parts there exists a dual figure dually constructed from its dual parts. A descriptively self-dual configuration is intrinsically self-dual if there exists a correlation which transforms each part into its dual part. Naturally the formal, descriptive, and intrinsic

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