THE ANALYTIC PROLONGATION OF A MINIMAL SURFACE

BY E. F. BECKENBACH

1. **Introduction.** A classical theorem is the following.[1]

THEOREM A. *If a minimal surface S cuts a plane Π orthogonally, then S is symmetric with respect to Π.*

We shall establish the following generalization of Theorem A.

THEOREM B. *If a minimal surface S is bounded in part by an arc C of a curve that lies in a plane Π, and if S approaches Π orthogonally, then S can be continued analytically as a minimal surface across Π and the extended surface is symmetric with respect to Π.*

A similar generalization of the following classical result has been given by J. Douglas:[2] *if a minimal surface contains a straight line in its interior, then the straight line must be an axis of symmetry of the surface.*

The proofs of Theorem A which have been given depend essentially on the fact that the plane curve is an interior curve on S. To prove Theorem B, it would be sufficient, in virtue of Theorem A, to prove that S can be continued analytically across Π. Actually, though, we establish both the possibility of analytic continuation and the symmetry at the same time, so that in particular a proof of Theorem A is included in our proof of Theorem B.

It is to be noted (see (2.3)) that we do not assume the arc C to be analytic. We prescribe the behavior of only one of the coordinate components, $z(u, v)$, as the parameter point approaches an arbitrary point on a given segment ab of the boundary of the domain of definition D. Indeed we are assuming not even that the part of the boundary of S in question is an arc of curve but only that the boundary lies in a plane.

It follows as a *consequence* of Theorem B, however, that the boundary of S on Π must necessarily be an analytic arc.

It is further to be noted (see (2.4)) that we do not assume that the normal to S approaches a definite position as the parameter point approaches a fixed point on ab but only that the component of the normal perpendicular to Π approaches zero.

2. **Analytic formulation.** We shall denote by D the upper half of the u, v-plane, $v > 0$; by D' the lower half, $v < 0$; and by ab a fixed open segment

Received August 26, 1941.

[1] See, for instance, the author's paper, *Minimal surfaces in Euclidean n-space*, American Journal of Mathematics, vol. 55(1933), pp. 458–468.

[2] J. Douglas, *The analytic prolongation of a minimal surface over a rectilinear segment of its boundary*, Duke Mathematical Journal, vol. 5(1939), pp. 21–29; see also Proc. Nat. Acad. Sci. U. S. A., vol. 26(1940), pp. 215–221.