AN APPLICATION OF THE CLASSICAL ORTHOGONAL POLYNOMIALS TO THE THEORY OF INTERPOLATION

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1. Introduction. Let

$$
\begin{array}{llll}
x_{11} & & & \\
x_{12} & x_{22} & & \\
\cdots & \cdots & \cdots & \cdots \\
x_{1,n} & x_{2,n} & \cdots & x_{n,n} \\
\cdots & \cdots & \cdots & \cdots
\end{array}
\qquad (a \leqq x_{1,n} < x_{2,n} < \cdots < x_{n,n} \leqq b) \tag{1}
$$

be a triangular matrix of abscissas on an interval $[a, b]$. Set $\omega_n(x) = c(x - x_{1,n})(x - x_{2,n}) \cdots (x - x_{n,n})$, with c an arbitrary non-zero constant. Then,

$$
l_{k,n}(x) = \frac{\omega_n(x)}{(x - x_{k,n})\omega_n'(x_{k,n})}, \quad \text{with} \quad l_{k,n}(x_{j,n}) = \delta_{kj} \qquad (k, j = 1, 2, \cdots, n; n = 1, 2, \cdots), \tag{2}
$$

are the fundamental polynomials of Lagrange interpolation of degree not exceeding $n - 1$ corresponding to the n-th set of abscissas. Let

$$
\begin{array}{llll}
y_{11} & & & \\
y_{12} & y_{22} & & \\
\cdots & \cdots & \cdots & \cdots \\
y_{1,n} & y_{2,n} & \cdots & y_{n,n} \\
\cdots & \cdots & \cdots & \cdots
\end{array} \tag{3}
$$

be a corresponding matrix of ordinates. Then, the Lagrange interpolation polynomial $L_n(x)$ has the property that

$$
L_n(x) = \sum_{k=1}^{n} y_{k,n} l_{k,n}(x), \qquad L_n(x_{j,n}) = y_{j,n} \qquad (j = 1, 2, \cdots, n; n = 1, 2, \cdots). \tag{4}
$$

In particular, if $f(x)$ is a function defined on the finite interval $[a, b]$ and if we select $y_{k,n} = f(x_{k,n})$ $(k = 1, 2, \cdots, n; n = 1, 2, \cdots)$, we write $L_n[f; x] \equiv L_n[f]$. The question whether $L_n[f] \to f(x)$ uniformly on $[a, b]$ as $n \to \infty$ for an arbitrary continuous $f(x)$ was settled definitively by Faber,[1] who showed that,

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[1] Jahresbericht der deutschen Mathematiker-Vereinigung, vol. 23(1914), pp. 192–210.