THE ABSOLUTE CONVERGENCE OF TRIGONOMETRICAL SERIES

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The purpose of this paper is to establish some theorems on absolute convergence of trigonometrical series. The first part of the paper is related to some classes of trigonometrical series which cannot converge absolutely at more than one point without being absolutely convergent everywhere. The second part deals with some properties of the sets of points at which a trigonometrical series can converge absolutely without being everywhere absolutely convergent. The third part is devoted to a generalization of the Denjoy-Lusin theorem.

In the first part of the paper, the following theorem is proved:

Theorem I. *If the series*

$$\sum \rho_n \cos (nx - \alpha_n) \qquad (\rho_n \geqq 0) \tag{1}$$

converges absolutely at two points x_0, x_1, *then the series* $\sum \rho_n \mid \sin n(x_1 - x_0) \mid$ *converges.*

This theorem, although very simple, does not seem to have been stated before, and it has some important consequences. It leads to the following theorems:

Theorem II. *The series* (1) *in which the sequence* $\{\rho_n\}$ *is non-increasing cannot converge absolutely at more than one point if* $\sum \rho_n = \infty$.

(Points whose abscissas differ from π are not considered as different.)

Theorem III. *The same theorem is true if instead of supposing the sequence* $\{\rho_n\}$ *non-increasing we suppose that* ρ_{n+p}/ρ_n *is bounded, independently of* n *and* $p > 0$.

Theorem IV. *The series* (1) *in which* $\sum \rho_n = \infty$ *cannot converge absolutely at more than one point if*

$$\sum_1^n \frac{1}{\rho_n} = O(n^2).$$

Theorem V. *If the assumptions on the coefficients* ρ_n *of any one of the Theorems* II, III, *or* IV *are satisfied, the series*

$$\sum \rho_n \cos (k_n x - \alpha_n) \qquad \left(\sum \rho_n = \infty\right) \tag{2}$$

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