AN ANALOGUE OF THE STAUDT-CLAUSEN THEOREM

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1. A set of rational functions B_m of an indeterminate x is defined by means of

$$\frac{t}{\psi(t)} = \sum_{m=0}^{\infty} \frac{B_m}{g_m} t^m,$$

where the several quantities involved are defined as follows. Put

$$[k] = x^{p^{nk}} - x,$$

$$F_k = [k][k-1]^{p^n} \cdots [1]^{p^{n(k-1)}}, \qquad F_0 = 1,$$

$$L_k = [k][k-1] \cdots [1], \qquad L_0 = 1;$$

the function in the denominator is given by[1]

$$\psi(t) = \sum_{k=0}^{\infty} \frac{(-1)^k}{F_k} t^{p^{nk}}.$$

Further for

$$m = \alpha_0 + \alpha_1 p^n + \cdots + \alpha_s p^{ns} \qquad (0 \leqq \alpha_i < p^n),$$

write

$$g_m = F_0^{\alpha_0} F_1^{\alpha_1} \cdots F_s^{\alpha_s}, \qquad g_0 = 1.$$

Thus B_m is defined for all $m \geqq 0$ and vanishes if m is not a multiple of $p^n - 1$. From the formula

$$\sum \frac{1}{E^m} = \frac{B_m}{g_m} \xi^m \qquad (p^n - 1 \mid m),$$

where the summation extends over all primary polynomials $E = E(x)$ with coefficients in $GF(p^n)$, and where

$$\xi = \lim_{k=\infty} \frac{(x^{p^n} - x)^{p^{nk}/(p^n-1)}}{L_k},$$

it follows that

$$B_m \neq 0 \qquad \text{for } p^n - 1 \mid m.$$

In discussing properties of B_m we therefore assume that m is a multiple of $p^n - 1$. Note that the coefficients involved in B_m lie in $GF(p)$, that is, are integers (mod p).

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[1] See this Journal, vol. 1(1935), pp. 137–168.