NECESSARY CONDITIONS IN GENERALIZED-CURVE PROBLEMS OF THE CALCULUS OF VARIATIONS

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In a preceding paper[1] we have developed the theory of generalized curves originated by L. C. Young.[2,3,4] For the problems of the calculus of variations in which a generalized curve is sought which minimizes an integral we have developed certain existence theorems. In a subsequent paper we shall obtain conditions under which the minimizing (generalized) curve can be shown to be an ordinary curve. But for this purpose we must develop the theory of the calculus of variations for generalized curves; and it is this development which forms the subject matter of the present paper. We shall develop necessary conditions for a minimum in problems of Bolza type.

Our extension of the theory must necessarily include, as a special case, the proof of the multiplier rule for rectifiable (ordinary) curves. Such a proof has already been given by Graves.[5] The non-parametric problem with generalized curves has been studied by L. C. Young;[6] however, Young considered only plane problems without side conditions, and we require a more extended theory. Moreover, we need a result not obtained by Young; it is important for us to know that for almost all t the partial derivatives F_{r^i} are constant over the set of vectors r carried by the minimizing curve C_0^* at $y_0(t)$. (Young obtained this result in a special case.[7]) Still further, in order to establish our existence theorems it is vital to know that the Weierstrass condition is satisfied along the minimizing curve, whether or not it is normal. This new form of the multiplier rule, including the Weierstrass condition, was first established in this note; the subsequent specialization to ordinary curves has already been published.[8]

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[1] E. J. McShane, *Generalized curves*, this Journal, vol. 6(1940), pp. 513–536; henceforth referred to by the letters GC.

[2] L. C. Young, *On approximation by polygons in the calculus of variations*, Proc. Royal Soc., (A), vol. 141(1933), pp. 325–341.

[3] L. C. Young, *Generalized curves and the existence of an attained absolute minimum in the calculus of variations*, Comptes Rendus de la Société des Sciences et des Lettres de Varsovie, Classe III, vol. 30(1937), pp. 212–234.

[4] L. C. Young, *Necessary conditions in the calculus of variations*, Acta Math., vol. 69(1938), pp. 229–258.

[5] L. M. Graves, *On the problem of Lagrange*, Amer. Jour. of Math., vol. 53(1931), pp. 547–554.

[6] Loc. cit. (footnote 4).

[7] Loc. cit. (footnote 4), pp. 248, 249.

[8] E. J. McShane, *On multipliers for Lagrange problems*, Amer. Jour. of Math., vol. 61(1939), pp. 809–819.