LINEAR FORMS AND POLYNOMIALS IN A GALOIS FIELD

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1. **Introduction.** Let $t, u_0, u_1, \cdots$ be indeterminates over the Galois field $GF(p^n)$, and put

$$(1.1) \qquad f_m(t) = f_m(t; u_0, \cdots, u_{m-1}) = \prod_{(c)} (t + c_0 u_0 + \cdots + c_{m-1} u_{m-1}),$$

the product extending over all sets $(c_0, \cdots, c_{m-1})$ in $GF(p^n)$. Then by a formula due to E. H. Moore[1] we may express $f_m(t)$ as the quotient of two determinants:

$$(1.2) \qquad f_m(t; u_0, \cdots, u_{m-1}) = \frac{D(u_0, \cdots, u_{m-1}, t)}{D(u_0, \cdots, u_{m-1})},$$

where

$$(1.3) \qquad D_m = D(u_0, \cdots, u_m) = \begin{vmatrix} u_0 & u_0^{p^n} & \cdots & u_0^{p^{nm}} \\ \cdots & \cdots & \cdots & \cdots \\ u_m & u_m^{p^n} & \cdots & u_m^{p^{nm}} \end{vmatrix},$$

and $D(u_0, \cdots, u_{m-1}, t)$ is defined by taking $u_m = t$.

In the special case[2]

$$(1.4) \qquad u_i = x^i \qquad (i = 0, 1, 2, \cdots),$$

where x is an indeterminate, the linear form

$$c_0 u_0 + c_1 u_1 + \cdots + c_{m-1} u_{m-1} \qquad (c_j \text{ in } GF(p^n))$$

reduces to $c_0 + c_1 x + \cdots + c_{m-1} x^{m-1}$, the general polynomial of degree $< m$, and $f_m(t)$ reduces to $\psi_m(t)$. We shall refer to (1.4) as the P-case. The chief object of the present paper is to extend certain properties of $\psi_m(t)$ to the more general $f_m(t)$. In particular we define an operator Ω^i for which

$$g(tw) = \sum \Omega^i g(w) f_i(t),$$

where $g(t)$ is an arbitrary linear[3] polynomial in t. Applications are made to the evaluation of certain power sums. The inverse of $f_m(t)$ and certain limiting cases are discussed briefly. Finally we define a polynomial $G_k(t)$ of degree k that reduces to $f_m(t)$ for $k = p^{nm}$.

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[1] Bulletin of the American Mathematical Society, vol. 2(1896), pp. 189–195. See also O. Ore, Transactions of the American Mathematical Society, vol. 35(1933), pp. 559–584; L. E. Dickson, Trans. Amer. Math. Soc., vol. 12(1911), p. 75.

[2] This Journal, vol. 1(1935), pp. 137–168. Cited as I.

[3] That is, of the form $\sum \beta_i t^{p^{ni}}$.