RESTRICTIONS IMPOSED BY CERTAIN FUNCTIONS ON THEIR FOURIER TRANSFORMS

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1. It is our purpose here to consider the restrictions imposed by the special behavior of a function on its Fourier transform. We shall consider two cases: (a) where the function has special behavior at infinity, and (b) where the function has special behavior at some finite point.

Case (a). The results here began with a suggestion of Wiener that both a function and its Fourier transform cannot be very small at infinity. This suggestion led to a theorem by Hardy,[1] a corollary of which is the fact that *if*

$$f(x) = O(|\,x\,|^n e^{-\frac{1}{2}x^2}) \qquad (x \to \pm\infty),$$

and if the Fourier transform of $f(x)$

$$g(u) = o(e^{-\frac{1}{2}u^2}) \qquad (u \to \pm\infty),$$

then $f(x) \equiv 0$.

This result is extended in a theorem of Morgan[2] who shows that *if*

$$f(x) = O(e^{-A|x|^p}) \qquad (x \to \pm\infty;\ p \geqq 2),$$

and its transform

$$g(u) = O(\exp\{-[A' + \epsilon]\,|\,u\,|^{p'}\}) \qquad (u \to \pm\infty),$$

where $\epsilon > 0$,

$$\frac{1}{p} + \frac{1}{p'} = 1,$$

and

$$A' = \frac{1}{p'(Ap)^{p'-1}} \sin \frac{\pi}{2(p-1)},$$

then $f(x) \equiv 0$.

The results we shall consider here, while obviously related to the above results, will differ from them in that first we shall restrict the behavior of $f(x)$ and $g(u)$ on only one side at infinity, for example, only as $x \to +\infty$ and $u \to +\infty$.

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[1] G. H. Hardy, *A theorem concerning Fourier transforms*, Journal London Math. Soc., vol. 8(1933), pp. 227–231.

[2] G. W. Morgan, *A note on Fourier transforms*, Journal London Math. Soc., vol. 9(1934), pp. 187–192.