

A COMPATIBLE INTEGRO-DIFFERENTIAL SYSTEM

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1. **Introduction.** This paper is concerned with the integro-differential system

$$(1.1) \quad L^i[u(x:s)] = \frac{\partial u^i(x:s)}{\partial x} + \Phi_j^i(x:s)u^j(x:s) + \int_\alpha^\beta K_j^i(x:s, t)u^j(x:t) dt,$$

$$(1.2) \quad U^i[u(x:s)] = \alpha_j^i(s)u^j(a:s) + \beta_j^i(s)u^j(b:s) \\ + \int_\alpha^\beta [A_j^i(s, r)u^j(a:r) + B_j^i(s, r)u^j(b:r)] dr.$$

$\Phi_j^i(x:s)$, $K_j^i(x:s, t)$, $\alpha_j^i(s)$, $\beta_j^i(s)$, $A_j^i(s, r)$, and $B_j^i(s, r)$ are to be known and continuous in the regions I_{xs} [$a \leq x \leq b$, $\alpha \leq s \leq \beta$], I_{xst} [$a \leq x \leq b$, $\alpha \leq s, t \leq \beta$], J_s [$\alpha \leq s \leq \beta$], and J_{sr} [$\alpha \leq s, r \leq \beta$]. We seek a set $u^i(x:s)$, continuous and with continuous first partial x derivatives in I_{xs} satisfying the non-homogeneous system

$$(1.3) \quad L^i[u(x:s)] = f^i(x:s), \quad U^i[u(x:s)] = X^i(s),$$

under the hypothesis that the homogeneous system

$$(1.4) \quad L^i[u(x:s)] = 0, \quad U^i[u(x:s)] = 0$$

is compatible of order k .

M. T. Hu¹ studied the incompatible case for a single equation and one boundary condition. §§2, 3, 4, and 5 consist largely of an extension to systems of equations of the results of his paper that are essential here. Since for the most part such generalizations are simple, we shall simply state results, elaborating only where the work is new or different.

Our main concern will be to devise generalized Green's functions $H_j^i(x, y:s)$ and $G_j^i(x, y:s, t)$ analogous to those for compatible differential systems developed by W. W. Elliott.² In addition we shall devise auxiliary functions $P_j^i(x:s)$ and $Q_j^i(x:s, t)$ such that the solution of (1.3), when it exists, may be written in the form

$$(1.5) \quad u^i(x:s) = \int_a^b H_j^i(x, y:s)f^j(y:s) dy + \int_a^b \int_\alpha^\beta G_j^i(x, y:s, t)f^j(y:t) dy dt \\ + P_j^i(x:s)X^j(s) + \int_\alpha^\beta Q_j^i(x:s, t)X^j(t) dt + A^i u_s^i(x:s).$$

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¹ M. T. Hu, *Linear integro-differential equations with a boundary condition*, Transactions of the American Mathematical Society, vol. 19(1918), pp. 363-407.

² W. W. Elliott, *Generalized Green's functions for compatible differential systems*, American Journal of Mathematics, vol. 50(1928), pp. 243-258.