A SET OF POLYNOMIALS

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1. Let $GF(p^n)$ denote a Galois (finite) field of order p^n. Let M denote a polynomial in an indeterminate x with coefficients in $GF(p^n)$:

$$M = M(x) = c_0 x^m + c_1 x^{m-1} + \cdots + c_m ;$$

for $c_0 \neq 0$, we write $\deg M = m$; for $c_0 = 1$, M is called primary. Further let[1]

$$\psi_m(t) = \prod_{\deg M < m} (t - M), \qquad \psi_0(t) = t, \tag{1.1}$$

where t is another indeterminate and the product extends over all M (including 0) of degree $< m$; then we have the formula

$$\psi_m(t) = \sum_{i=0}^{m} (-1)^{m-i} \begin{bmatrix} m \\ i \end{bmatrix} t^{p^{ni}}, \tag{1.2}$$

where

$$\begin{bmatrix} m \\ i \end{bmatrix} = \frac{F_m}{F_i L_{m-i}^{p^{ni}}}, \qquad \begin{bmatrix} m \\ 0 \end{bmatrix} = \frac{F_m}{L_m}, \qquad \begin{bmatrix} m \\ m \end{bmatrix} = 1, \tag{1.3}$$

and

$$\begin{aligned} F_m &= [m][m-1]^{p^n} \cdots [1]^{p^{n(m-1)}}, & F_0 &= 1, \\ L_m &= [m][m-1] \cdots [1], & L_0 &= 1, \\ [m] &= x^{p^{nm}} - x. \end{aligned} \tag{1.4}$$

We remark that

$$\psi_m(x^m) = \psi_m(M) = F_m ,$$

for M primary of degree m, so that F_m is the product of the primary polynomials of degree m.

For k an arbitrary integer $\geqq 0$, put

$$k = \alpha_0 + \alpha_1 p^n + \cdots + \alpha_s p^{ns} \qquad (0 \leqq \alpha_i < p^n), \tag{1.5}$$

and define the polynomial g_k by means of

$$g_k = F_1^{\alpha_1} \cdots F_s^{\alpha_s}, \qquad g_0 = 1. \tag{1.6}$$

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[1] See this Journal, *On certain functions connected with polynomials in a Galois field*, vol. 1(1935), pp. 137–168, p. 141. This paper will be cited as I.