A GENERALIZATION OF POISSON'S SUMMATION FORMULA

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Poisson's formula. The standard form of Poisson's formula is[1]

$$\sum_{-\infty}^{\infty} f(m) = \sum_{-\infty}^{\infty} F(2\pi n), \tag{1}$$

$$F(\alpha) = \int_{-\infty}^{\infty} e^{-i\alpha x} f(x)\, dx. \tag{2}$$

If $f(z)$ is analytic in a strip

$$| y | < y_0 \tag{3}$$

of the complex plane $z = x + iy$, $\sum f(m)$ is the sum of the residues of the function

$$\pi f(z) \frac{\cos \pi z}{\sin \pi z} \tag{4}$$

and therefore it is the limit, as $T \to \infty$, of the Cauchy integral of the function (4) around the rectangle with the corners $\pm T \pm bi$ ($b < y_0$). In order to transform the integral into $\sum F(2\pi n)$ we have to replace $-i \cot \pi z$ by the expansion

$$1 + 2 \sum_{n=1}^{\infty} e^{-2n\pi i z} \tag{5}$$

for $y < 0$ and by

$$-(1 + 2 \sum_{n=1}^{\infty} e^{2n\pi i z}) \tag{6}$$

for $y > 0$.[2]

In our generalizations we will take an unspecified meromorphic function $\varphi(z)$ in a strip (3) instead of the particular function $\cot \pi z$. This will lead to a formula

$$\sum_{-\infty}^{\infty} r_m f(a_m) = \int_{-\infty}^{\infty} F(\alpha)\, d\Phi(\alpha). \tag{7}$$

The numbers a_m will be simple poles of $\varphi(z)$ and r_m their residues, and the weight function $\Phi(\alpha)$ will be taken from general expansions, analogous to (5) and (6), of the function $\varphi(z)$ in two strips in which it has no poles.

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[1] Compare S. Bochner, *Fouriersche Integrale*, p. 33; E. C. Titchmarsh, *Fourier Integrals*, p. 60.

[2] Compare E. Lindelöf, *Calcul des Résidus*, Chapter III.