EXTENDING MAPS OF PLANE PEANO CONTINUA

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1. **Introduction.** When can a homeomorphism T of a Peano continuum M on a sphere S to a set M' on a sphere S' be extended to a homeomorphism T' which carries the whole sphere S into S'? Gehman has solved this problem for the extension of T to a map of a *plane* containing such a space M.[1] His condition can easily be modified to include the case of M and M' on spheres.[2] But this condition, which requires that certain "sides" of M be preserved by T, is extrinsic to the given M, and requires in fact that one establish the existence of certain arcs of S and S' which may cut through M and its complement $S - M$ in a complicated fashion. We establish here another necessary and sufficient condition for the extendibility of T. This condition is intrinsic, and applies only to the triods of M, where by a *triod* in M we mean a set $\tau = [\alpha, \beta, \gamma]$ of three arcs of M with a common end point, the vertex of the triod, such that any two of these arcs intersect only in this vertex.

Theorem 1. *If M and M' are topologically equivalent Peano continua lying respectively on spheres S and S', then a homeomorphism T of M to M' can be extended to a homeomorphism T' of S to S' if and only if T preserves the relative sense of every pair of triods of M.*

To say that T *preserves the relative sense of triods* here means that T carries any two triods τ_1 and τ_2 of M which have the same sense on S into two triods τ_1' and τ_2' which have the same sense (i.e., both are clockwise or both are counterclockwise) on S'. The precise method for treating this concept of "sense" is sketched in §2.

This theorem implies that when T preserves sense on triods, it necessarily carries each complementary domain boundary (c.d.b.) of M into a c.d.b. of M'. An extendibility condition for planes may thus be found by projecting the sphere from a point in a suitable complementary domain.

Theorem 2. *A homeomorphism $T(M) = M'$ between two plane Peano continua M and M' can be extended to the whole planes if and only if T preserves the*

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[1] H. M. Gehman, *On extending a continuous* (1-1) *correspondence of two plane continuous curves to a correspondence of their planes,* Transactions of the American Mathematical Society, vol. 28(1926), pp. 252–265.

[2] V. W. Adkisson, *On extending a continuous* (1-1) *correspondence of continuous curves on a sphere,* Comptes Rendus des Séances de la Société des Sciences et des Lettres de Varsovie, vol. 27(1934), pp. 5–9.

H. M. Gehman, *On extending a homeomorphism between two subsets of spheres,* Bulletin of the American Mathematical Society, vol. 42(1936), pp. 79–81.