SOME PROPERTIES OF ${}_3F_2(-n, n+1, \zeta; 1, p; v)$

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The fact that the Legendre function $P_n(x)$ may be expressed as the hypergeometric series

$$P_n(x) = F\left(-n, n+1; 1; \frac{1-x}{2}\right) = \sum_{r=0}^{\infty} \frac{(-n)_r(n+1)_r}{r!r!}\left(\frac{1-x}{2}\right)^r,$$

where $(\alpha)_r = \alpha(\alpha+1)\cdots(\alpha+r-1)$, together with the fact that the generalized hypergeometric functions, studied by Bateman and Pasternack,

$$F_n(z) = {}_3F_2\left(-n, n+1, \frac{1+z}{2}; 1, 1; 1\right) = \sum_{r=0}^{\infty} \frac{(-n)_r(n+1)_r\left(\frac{1+z}{2}\right)_r}{r!r!r!},$$

$$F_n^m(z) = {}_3F_2\left(-n, n+1; \frac{1+z+m}{2}; 1, m+1; 1\right)$$

$$= \sum_{r=0}^{\infty} \frac{(-n)_r(n+1)_r\left(\frac{1+z+m}{2}\right)_r}{r!r!(m+1)_r}$$

have many interesting properties, suggests that other hypergeometric series in which $(-n)_r(n+1)_r/(r!\,r!)$ appears as a factor in the general term may also be of interest.

Here we examine one of these series, namely,

$$H_n(\zeta, p, v) = {}_3F_2(-n, n+1, \zeta; 1, p; v)$$

$$= \sum_{r=0}^{\infty} \frac{(-n)_r\,(n+1)_r\,(\zeta)_r}{r!r!(p)_r} v^r.$$

When the generalizations given here are compared with the earlier results, only those of Bateman's involving $F_n(z)$ will be mentioned, although in many cases Pasternack [9][1] has obtained relations which, from the standpoint of generality, lie between those given here and those given by Bateman. This omission is made for the sake of brevity and simplicity. In all of the following work we assume that p is not a negative integer, and that n, unless otherwise stated, is a positive integer so that $H_n(\zeta, p, v)$ is a polynomial of degree n; and we shall omit the subscripts 3 and 2 on the hypergeometric function ${}_3F_2$ when it is convenient to do so.

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[1] Numbers in brackets refer to the bibliography.