INTEGERS WHICH ARE NOT REPRESENTED BY CERTAIN TERNARY QUADRATIC FORMS

By R. D. James

1. **Introduction.** It is well known that all integers except those of a specified form are represented by certain ternary quadratic forms. Thus, an integer N is represented by the form

$$x^2 + y^2 + z^2$$

if and only if N is not of the form $4^k(8l + 7)$. In this paper we shall show that all sufficiently large integers just fail to be represented by a ternary quadratic form. More precisely, we prove the following[1]

Theorem. *Let any negative integer d be written as $-2^b S^2 h$, where S and h are odd, and h is quadratfrei. Then, every sufficiently large integer N can be represented in the form*

$$N = u^2 + epH^2M, \tag{1.1}$$

where u is a positive integer, p is a prime such that $(d \mid p) = -1$, every prime dividing H also divides d, every prime q dividing M satisfies $(d \mid q) = 1$, and e is 1 except in the three following cases:

(1.21) b *odd*, $h \mid N$, $h \equiv 3 \pmod 4$, $N \equiv 6 \pmod 8$,

(1.22) b *even*, $h \mid N$, $h \equiv 1 \pmod 4$, $N \equiv 2 \pmod 4$,

(1.23) b *odd*, $h \mid N$, $h \equiv 1 \pmod 4$, $N \equiv 2 \pmod 8$,

in all of which $e = 2$.

It is the presence of the single prime p in (1.1) which prevents N from being represented by a ternary quadratic form. For, if $(d \mid p)$ were equal to $+1$, the product pH^2M would be represented by some binary quadratic form of discriminant d.[2] For example, if $d = -3$ the p in (1.1) would be congruent to 2 (mod 3), $H = 3^k$, $k \geqq 0$, and every prime q dividing M would be congruent to 1 (mod 3). The binary quadratic form in question is

$$v^2 + vw + w^2.$$

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[1] The author is indebted to the referee for helpful suggestions which led to the proof of the theorem in its present general form.

[2] L. E. Dickson, *Introduction to the Theory of Numbers*, Chapter V.