A UNIQUENESS THEOREM FOR ANALYTIC ALMOST-PERIODIC FUNCTIONS

By S. Bochner

We start from the following theorem which can be proved along familiar lines.

(i) If the functions $F_n(z)$ $(n = 1, 2, \cdots)$ are each analytic in the annulus $a < r < b$ and (uniformly) continuous in the closure $a \leqq r \leqq b$,

(ii) if there exists a constant A such that for $a \leqq r \leqq b$ and $n = 1, 2, \cdots$

$$\frac{1}{2\pi} \int_0^{2\pi} | F_n(re^{i\theta}) | \, d\theta \leqq A,$$

and

(iii) if there exists an interval $\theta_0 < \theta < \theta_1$ for which

$$\lim_{n \to \infty} \frac{1}{2\pi} \int_{\theta_0}^{\theta_1} | F_n(ae^{i\theta}) | \, d\theta = 0$$

or, what is the same, if there exists a non-negative continuous periodic function $\varphi(\theta)$, which does not vanish identically, for which

$$\lim_{n \to \infty} \frac{1}{2\pi} \int_0^{2\pi} | F_n(ae^{i\theta}) | \, \varphi(\theta) \, d\theta = 0,$$

then the sequence of functions $F_n(z)$ converges towards 0 everywhere in the annulus.

Putting $z = e^s$, $s = \sigma + it$, we will generalize this theorem to analytic almost-periodic functions in strips.[1] The periodic function $\varphi(\theta)$ will be replaced by an (uniformly continuous) almost-periodic function of the real variable t. The mean value

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \chi(t) \, dt$$

of an almost-periodic function $\chi(t)$ will be denoted by

$$M_t \chi(t).$$

Theorem. (i) *If each function* $f_n(s)$ $(n = 1, 2, \cdots)$ *is analytic in the strip*

(1) $$\alpha < \sigma < \beta$$

and uniformly continuous and almost periodic in the closed strip

(2) $$\alpha \leqq \sigma \leqq \beta,$$

Received September 13, 1939.

[1] See A. S. Besicovitch, *Almost Periodic Functions*, Cambridge, 1932.