LIMIT POINTS OF SEQUENCES AND THEIR TRANSFORMS BY METHODS OF SUMMABILITY

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1. **Introduction.** Let $\{s_n\}$ be a complex sequence, and let A be its set of limit points. The problem which we consider is that of determining, for each of several methods of summability, whether the set of limit points of the transform of each bounded sequence $\{s_n\}$ is a connected set.

In §2 examples are given which show that if no restrictions are placed on the complex sequence $\{s_n\}$, then the set A of its limit points need not be connected. In §3 sufficient conditions that A be connected are given. In §4 theorems concerning transforms of bounded complex sequences and their sets of limit points are proved. We devote §§5, 6, 7, 8, and 9 to the Hölder, Cesàro, Riesz, de la Vallée Poussin, and Euler transforms, respectively, of bounded complex sequences $\{s_n\}$. In each case we determine whether the set of limit points of the transform of $\{s_n\}$ is connected.

2. **Some examples.** Since A is a closed set, to say that it is connected means that A cannot be broken up into two mutually exclusive subsets A_1 and A_2, both closed, unless either $A_1 = \Lambda$ (empty set) or $A_2 = \Lambda$.

To show that A need not be connected, let us consider the following three complex sequences:

$$1, i, 1, i, 1, i, \cdots, \tag{2.1}$$

$$1, i, 2, 1, i, 3, 1, i, 4, \cdots, \tag{2.2}$$

$$\begin{gathered} 0, \frac{i}{10}, \frac{2i}{10}, \cdots, i, \frac{1}{10} + i, \frac{2}{10} + i, \cdots, 1 + i, 1 + \frac{9i}{10}, 1 + \frac{8i}{10}, \cdots, 1, \\ 1 + \frac{i}{20}, 1 + \frac{2i}{20}, \cdots, 1 + 2i, \frac{19}{20} + 2i, \frac{18}{20} + 2i, \cdots, 2i, \frac{39i}{20}, \cdots, 0, \\ \frac{i}{30}, \frac{2i}{30}, \cdots, 3i, \frac{1}{30} + 3i, \frac{2}{30} + 3i, \\ \cdots, 1 + 3i, 1 + \frac{89i}{30}, 1 + \frac{88i}{30}, \cdots, 1, \\ 1 + \frac{i}{40}, 1 + \frac{2i}{40}, \cdots, 1 + 4i, \frac{39}{40} + 4i, \frac{38}{40} + 4i, \\ \cdots, 4i, \frac{159i}{40}, \frac{158i}{40}, \cdots, 0, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots . \end{gathered} \tag{2.3}$$

Received November 8, 1938.