THE PROBLEM OF TYPE FOR A CERTAIN CLASS OF RIEMANN SURFACES

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1. **Introduction.** In the present paper there are obtained sufficient conditions that a certain class of open simply-connected Riemann surfaces shall be of parabolic type,[1] that is, can be mapped one-to-one and in general conformally on the plane with one point removed. The surfaces to be considered are, briefly, surfaces which have a single transcendental singularity which is a limit point of algebraic branch points of first order, and a logarithmic branch point.

The results are derived from a criterion due to Ahlfors.[2] It can be stated as follows. Let the open simply-connected surface W be spread out over the w-plane and a metric on W be defined by a differential form

$$d\sigma = \lambda(u, v) \mid dw \mid, \qquad w = u + iv,$$

where λ is a real, single-valued function, continuous on W with the exception of certain isolated points. Moreover, let the metric be so chosen that no singularity of the surface is accessible along a path of finite length. Let W_ρ be the region of the surface consisting of those points whose distance from a certain initial point P_0, in the metric considered, does not exceed a positive number ρ. Let $L(\rho)$ be the length of the boundary of W_ρ in the metric considered. Then, *a necessary and sufficient condition that W be of parabolic type is that there exist a metric of the type defined above such that the integral*

$$\int^{\infty} \frac{d\rho}{L(\rho)}$$

diverges.

2. **Class of surfaces to be considered.** The surfaces W to be considered are of the following sort.

Let $\{A_\nu\}$ $(\nu = 1, 2, 3, \cdots)$ be a countably infinite set of points of the real axis of the w-plane, which has as sole limit point the point at infinity. Moreover, suppose $A_\nu > 0$ for ν odd and $A_\nu < 0$ for ν even. Over each point of $\{A_\nu\}$ shall lie one and only one algebraic branch point of first order. There shall be no other algebraic branch points. There shall be a logarithmic branch point over $w = 0$ along with an infinite number of smooth sheets. There will

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[2] Comptes Rendus, vol. 201(1935), pp. 30–32.