THE INVARIANT THEORY OF THE TERNARY TRILINEAR FORM

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1. **Introduction.** Let x, y, z be three digredient ternary variables represented by points x, y, z in three respective planes E_x, E_y, E_z. We consider the trilinear form $F(x, y, z) = (\alpha x)(\beta y)(\gamma z) = \sum_{h,i,j=1}^{3} a_{hij}x_h y_i z_j$, where the a_{hij} are arbitrary complex numbers, and where we suppose that the form can be expressed in no fewer variables. The form has 27 coefficients, or 26 projective constants, of which we may eliminate $3 \cdot 8$ by proper projective transformations in E_x, E_y, E_z. For a form thus involving only 2 absolute projective constants, a complete discussion and classification of types may be anticipated.

In a joint article by R. M. Thrall and the author [12][1] the classification of such forms was given under (i) non-singular linear transformations on the sets of variables taken separately, and (ii) interchanges of the sets of variables. Two forms equivalent under (i) were called equivalent; if equivalent under (i) and (ii), they were called generally or g-equivalent. In a previous article [11] Thrall classified analogous forms in a $GF(p)$. For other references to recent studies involving the group-theoretic point of view [12] and [11] may be consulted. In [12] the approach was geometric; we studied the cubics $X(x) = 0$, $Y(y) = 0$, $Z(z) = 0$ obtained by equating to zero the determinants $| M_x(x) |$, $| M_y(y) |$, $| M_z(z) |$, where, e.g., $M_x(x)$ is the matrix $(\sum_h a_{hij}x_h)$. Particular attention was paid to the cases where the cubics had singular points or degenerated; for all such cases canonical forms for F were obtained. While the case of the elliptic cubic was discussed briefly from the analytic viewpoint, the method of attack made it impractical to set up a canonical form. In the present paper such a form is determined algebraically (see §4).

A promising method of investigation is the study of the concomitants of F. While certain of the concomitants have been previously studied, they have appeared rather in connection with bilinear than with trilinear forms. For example, C. Jordan [4] investigates the reducibility of the linear system of bilinear forms given by the trilinear form $T_{lmn} = \sum a_{\alpha\beta\gamma}\lambda_\alpha\mu_\beta x_\gamma$. For $l = m = n = 3$, he bases his discussion of the form T_{lmn} on the position of λ with respect to the cubic in λ given by the discriminant of the net of bilinear forms in μ, x. He does not mention the fact that this cubic is only one of three corresponding to our $X(x) = 0$, etc., which play equally important rôles so far as T_{lmn} is concerned. On the other hand, when J. Rosanes [8] studies the 3

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[1] Numbers in brackets refer to the bibliography at the end of the paper.