# THE VARIATION OF THE SIGN OF $V$ FOR AN ANALYTIC FUNCTION $U + iV$

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1. **Introduction.** The following theorem is M. Cartwright's [2]:[1]

Theorem A. *Let*

$$f(z) = \sum_0^\infty a_n z^n$$

*be regular and multivalent of order $p$ for $|z| < 1$ and have $q$ zeros within this circle. Then for $r < 1$*

$$|f(re^{i\theta})| < A(p)\mu_q(1 - r)^{-2p},$$

*where $A(p)$ is a constant depending only upon $p$, and where*

$$\mu_q = \max\{|a_0|, |a_1|, |a_2|, \cdots, |a_q|\}.$$

Recently, with the help of the preceding theorem, M. Biernacki [1] established the inequality for the coefficients of $f(z)$ given by the following theorem.

Theorem B. *With the same hypotheses as in Theorem* A, *the coefficients of $f(z)$ satisfy the inequality*

$$|a_n| < A(p)\mu_q n^{2p-1} \qquad (n > 0),$$

*where*

$$\mu_q = \max\{|a_1|, |a_2|, \cdots, |a_q|\}.$$

The following new theorems established in this paper appear to give somewhat similar inequalities under a different hypothesis. The theorems to follow below overlap Theorems A and B in some cases, especially when $f(z)$ is real on the real axis.

Let $f(re^{i\theta}) = U(r, \theta) + iV(r, \theta)$. Let $z = re^{i\theta}$ traverse the circle $|z| = r$ once, starting at any point $z_0 = re^{i\theta_0}$ where $V(r, \theta_0) \neq 0$. If $r < 1$, $V(r, \theta)$ is a continuous function of $\theta$. As $z$ makes a complete revolution around the circle from $z_0$ and back again to $z_0$, $V(r, \theta)$ has either (i) a constant sign or (ii) changes sign an even number of times, if we assume that the number of changes in sign is finite. We now state the theorems to be demonstrated in this paper.

Theorem 1. *Let*

$$f(z) = \sum_0^\infty a_n z^n = U + iV$$

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[1] Numbers in brackets refer to the list of references at the end of the paper.