# INVARIANTS

By Hermann Weyl

The *theory of invariants* came into existence about the middle of the nineteenth century somewhat like Minerva: a grown-up virgin, mailed in the shining armor of algebra, she sprang forth from Cayley's Jovian head. Her Athens over which she ruled and which she served as a tutelary and beneficent goddess was *projective geometry*. From the beginning she was dedicated to the proposition that all projective coördinate systems are created equal. Indeed, at that time the viewpoint of projective invariance was the one universally accepted in geometry. The rise of projective geometry had first been brought about by truly geometric stimuli, the study of conic sections, the theory of perspective and by the development of descriptive geometry, and the so-called synthetic direction of Steiner and von Staudt has confirmed the fertility of the projective attitude with respect to pure geometry.

However, its gaining such immense preponderance was, if I am not mistaken, due to algebraic rather than geometric reasons: namely, to the fact that the group of projectivities is expressed by the simplest of all continuous groups, the group of all homogeneous linear transformations. Plücker in the preface of his first work (*Analytisch-geometrische Entwicklungen*, vol. 1, 1828) openly espoused the ascendency of algebra, or, as he said, analysis, over geometry. So that perhaps one had better speak of geometric algebra than of algebraic geometry, namely, of an algebra which, in establishing its theorems and in the search for the proofs thereof, uses geometric terms and is guided by geometric intuition. The modern evolution, as far as it does not point its needle toward topology, has on the whole been marked by a trend of algebraization, notwithstanding the undeniable merits of the great school of Italian geometers.

The dictatorial rule of the projective idea in geometry was first successfully broken by the German astronomer and geometer Möbius. One is forced to realize that the group of all homogeneous linear transformations is not the only one worthy of consideration and capable of serving as the group of automorphisms in a geometric space. Möbius does not yet possess the general idea of a group; however, his notion of *Verwandtschaft* meets the same purpose in each special case he considers. The universal group theoretic interpretation was first

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No explicit references to the literature were given in the address; they can readily be supplied from the author's book *The Classical Groups, their Invariants and Representations*, Princeton, 1939. For the general foundations of the theory of invariants compare in particular v.d. Waerden, Mathematische Annalen, vol. 113(1936), pp. 14–35.