ON BERNOULLI'S NUMBERS AND FERMAT'S LAST THEOREM (SECOND PAPER)

BY H. S. VANDIVER

1. Further examination of Fermat's Last Theorem for special exponents. In the first paper under the present title[1] the writer gave some of the details of the computations which resulted in the proof of Fermat's Last Theorem for all prime exponents l such that $307 < l < 617$, with the exception of 587. At the end of the paper it is stated that the work has been carried out for 587 and since the criteria are found to hold, the theorem is proved for that exponent. The details are as follows. As noted in B.F. (p. 576) the numbers in the set

$$B_1, B_2, \cdots, B_{\frac{1}{2}(l-3)} \tag{1}$$

which are divisible by l when $l = 587$ are B_{45} and B_{46}, so that 587 is irregular and, as in the treatment of irregular primes in B.F., we employ Theorem 1 of that paper which we repeat here for easy reference:

THEOREM 1. *Under the assumptions: none of the units* E_a $(a = a_1, a_2, \cdots, a_s)$ *is congruent to the l-th power of an integer in* $k(\zeta)$ *modulo* $\mathfrak{p}$, *where* $\mathfrak{p}$ *is a prime ideal divisor of* p; p *is a prime* $< (l^2 - l)$ *of the form* $1 + lk$; *and* $a_1, a_2, \cdots, a_s$ *are the subscripts of the B's in the set* (1) *which are divisible by* l; *the relation*

$$x^l + y^l + z^l = 0 \tag{2}$$

is impossible in non-zero integers x, y and z, if l is a given odd prime, and

$$E_n = \prod_{i=0}^{\frac{1}{2}(l-3)} \epsilon(\zeta^{r^i})^{r^{-2in}},$$

$$\epsilon = \left(\frac{(1 - \zeta^r)(1 - \zeta^{-r})}{(1 - \zeta)(1 - \zeta^{-1})}\right)^{\frac{1}{2}},$$

r being a primitive root of l and $\zeta = e^{2i\pi/l}$.

Applying this to the case $l = 587$, we find for $r = -10$, $d = 2^{14}$, $p = 8219$, $\rho = 2$ and $n = 45$, ind $E_n(d) \equiv 576 \pmod{587}$ and for $n = 46$, ind $E_n(d) \equiv 60 \pmod{587}$. Here, as in B.F., d is an integer such that $d^l \equiv 1 \pmod{p}$ and ρ is a primitive root of p. Since ind $E_n(d) \not\equiv 0 \pmod{l}$ in the above, the criteria of the theorem are satisfied and Fermat's Last Theorem is proved for $l = 587$.

As noted in B.F. (p. 576) the prime 617 is irregular and B_{10}, B_{87} and B_{169} constitute all the B's in the set (1) which are divisible by l. Then applying Theorem 1, we find for $r = 410$, $d = 3^8$, $p = 4937$, $\rho = 3$, ind $E_{10}(d) \equiv 55$;

Received January 30, 1939.

[1] This Journal, vol. 3(1937), pp. 569–584. This paper will be referred to here as B.F.