PRESERVATION OF PARTIAL LIMITS IN MULTIPLE SEQUENCE TRANSFORMATIONS

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1. **Introduction.** Problems (iii) and (iv) in §1.4 of[1] H_1 were solved in that paper only for $s_{k^2} = 0$. It is the purpose of the present paper to give complete solutions. Existence of the transform σ_m is assumed for each m.

2. **Additional notations and definitions.** Let X and Y denote classes of sequences. The notation X $\rightarrow$ Y *row reg* shall signify row regularity of the transformation X $\rightarrow$ Y in case each of X and Y is the class of regularly convergent sequences, and ultimate row regularity in all other cases in question. (See §1.4 of H_1.) Thus[2] NS RC $\rightarrow$ RC *row reg* will mean NS RC $\rightarrow$ RC with $\sigma_{k^2} = s_{k^2}$ for all k^2. The notation NS RC $\rightarrow$ RC *ul row reg* shall mean NS RC $\rightarrow$ RC with $\sigma_{k^2} = s_{k^2}$ for all k^2 sufficiently large.

Consider the matrix $|| b_{mk} ||$, where $b_{mk} = a_{mk}$ $(k \neq m)$, and $b_{mm} = a_{mm} - 1$. Define the sequence $\{\tau_m\}$ by the equations

$$\tau_m = \sum_{k=1}^{\infty} b_{mk} s_k \equiv \sigma_m - s_m,$$

and let NS X-* $\rightarrow$ Y denote NS $\{\tau_m\}$ be of class Y whenever $\{s_k\}$ is of class X.

3. **List of theorems (first form).** The following theorems are obvious.

NS RC, NS BURC, NS URC $\rightarrow$ URC row reg are, respectively, NS RC, NS BURC, NS URC- $\rightarrow$ URCRN.*

NS RC, NS BURC $\rightarrow$ BURC row reg are, respectively, NS RC, NS BURC- $\rightarrow$ BURCRN.*

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NS BURC, NS URC $\rightarrow$ RC row reg are, respectively, NS BURC, NS URC $\rightarrow$ RC, and, respectively, NS BURC, NS URC- $\rightarrow$ URCRN.*

NS RC $\rightarrow$ RC row reg are NS RC- $\rightarrow$ RCRN.*

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[1] H_1 will denote the author's paper, *Transformations of multiple sequences*, this Journal, vol. 2(1936), pp. 29–60. The present paper assumes familiarity with the contents of H_1, the ideas, terminology, notations, and results of which are freely used without further comment.

[2] NS shall abbreviate *conditions necessary and sufficient that.*