SURFACES OF NEGATIVE CURVATURE AND PERMANENT REGIONAL TRANSITIVITY

By Anna Grant

1. **Introduction.** The various problems connected with transitivity have been treated extensively for the flows defined by the geodesics on two-dimensional manifolds of negative curvature. A description of the extent to which solutions of the problems have been attained has been given by Hedlund [7].[1]

The manifolds in question can be obtained by identifying the points congruent under a Fuchsian group. *The present paper shows that if the Fuchsian group is of the first kind and the manifold is of negative curvature, the property of permanent regional transitivity holds.* That is, the geodesics define a flow in the space of elements such that if O is any open set of elements at time t_0, O_t is the image of O after time t, and O^* is any other open set of elements, there exists a $\bar{t}$ such that for $|t| > \bar{t}$ the set $O_t \cdot O^*$ is not empty. It is thus an extension of a similar result obtained by Hedlund [6] in the case of *constant* negative curvature. The extension requires the derivation of numerous geometric results which should be useful in the further study of the geodesic flows on the surfaces under consideration.

2. **A class of simply-connected two-dimensional manifolds.** Let U denote the unit circle $u^2 + v^2 = 1$, and let Ψ be its interior, with the following metric defined in Ψ:

$$ds^2 = \frac{\lambda^2(u, v)(du^2 + dv^2)}{(1 - u^2 - v^2)^2}, \tag{2.1}$$

$\lambda(u, v)$ of class C^m, $m \geqq 5$, and $0 < a \leqq \lambda(u, v) \leqq b$ in Ψ. The *length* of any curve segment of class C' in Ψ is $\int ds$ evaluated over the curve, ds given by (2.1). The geodesics defined by (2.1) are of at least class C^2 in arc length, coordinates of initial point, and initial direction. The term *geodesic* will refer to the geodesics defined by (2.1). Given a point in Ψ and direction at this point, there is a unique geodesic passing through the given point in the given direction.

If $\lambda(u, v) \equiv 2$ in Ψ, the geodesics are arcs of circles orthogonal to U and are called *hyperbolic lines.* Given any two points P and Q in Ψ, there is a unique hyperbolic line segment joining them; and $\int ds$ evaluated over this segment,

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[1] Numbers in brackets refer to the bibliography at the end of the paper.