THE MAPS DETERMINED BY THE PRINCIPAL CURVES ASSOCIATED WITH FIVE AND SIX POINTS IN THE PLANE

By C. Ronald Cassity

1. Introduction

Any set of n generally chosen points of the plane determines a set of principal curves (P-curves). This set is finite in number if $n < 9$. The P-curves associated with the points divide the projective plane into regions and thus form a *map* on the plane. Such a map will be described by telling what kinds of regions it contains, how many of each kind, and in what manner they fit together. Two maps will be considered as identical if and only if the description of one is also a description of the other.

The formation of a map by the division of the projective plane into twelve five-sided regions by the P-curves attached to four generally chosen points in the plane has been studied by Coble and Brahana [1].[1] The group associated with the figure is derived and the geometrical interpretation of its various types of elements discussed. The order of the group is 120. Each of its elements can be expressed algebraically, for they are either collineations or quadratic transformations. The map has the maximum possible amount of *regularity*, for every element of the group can be interpreted as a topological transformation of the map into itself.

The cases $n = 5$ and $n = 6$ lead, by mappings by the linear system of cubic curves on the base points, to the rational surface of order four in S_4 and to the general cubic surface in S_3, respectively. The P-curves associated with the points map into lines on the surfaces. Hence the maps determined by the P-curves in the plane are identical with those determined by the lines on the surfaces. It is the author's purpose to discuss these maps in detail. In particular, it will be shown that, whereas the map determined by four points has *complete* regularity, that determined by five points has only *partial* regularity and that determined by six points has *no* regularity. The total lack of regularity in the map on the cubic surface may be somewhat surprising to those familiar with the results obtained by Klein ([8], p. 570) and Zeuthen ([12], p. 8). Their descriptions of the map are correct, but are incomplete according to the definition given above.

§3.7 is devoted to the correction of an error made by Zeuthen ([12], p. 8).

2. The case $n = 5$

Five points of the plane are, for present purposes, "generally chosen" if no two are coincident and no three collinear. Associated with such a set of five

Received October 4, 1938.

[1] Numbers in brackets refer to the bibliography at the end of the paper.