SURFACES IN FOUR-SPACE OF CONSTANT CURVATURE

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1. **Introduction.** We shall divide the work into two parts: (A) ruled surfaces V_2 imbedded in four-space of constant curvature S_4; (B) surfaces V_2 imbedded in four-space of constant curvature S_4 and such that the normal curvature locus of V_2 is linear.[1]

In (A) we classify the ruled V_2 in S_4 by means of the normal curvature locus. Two possible cases exist: (1) the normal curvature locus consists of axial points; (2) this locus consists of planar points. If the locus is axial, then by Struik's extension of Segre's theorem,[2] these V_2 are either ruled V_2 in S_3 or developable V_2 in S_4. If the locus is planar, then we show a one-to-one correspondence exists between any ruled V_2 in S_3 and a set of ruled V_2 in S_4, where S_3 and S_4 both have the same curvature K.[3]

In (B) we shall discuss a class of surfaces V_2 in an S_4 of constant curvature K which can be placed into a one-to-one isometric correspondence with any V_2 in an S_3 of constant curvature $K + L^2$.

Finally, we shall show that correspondence theorems of the type mentioned here furnish us with a method of giving existence proofs.

2. **Notation.** In an S_4 we introduce the coördinate system

$$y^\kappa \qquad (\kappa, \lambda, \mu = 1, 2, 3, 4). \tag{2.1}$$

By means of the equations

$$y^\kappa = y^\kappa(u^a) \qquad (a, b, c = 1, 2) \tag{2.2}$$

containing the two essential parameters u^1, u^2, we introduce a two-dimensional manifold in S_4. If the tangent two-dimensional planes E_2 of the surface do not cut the null cone of S_4 in more than a finite number of lines at any point of the surface, then a Riemannian metric is induced in the surface and it can be called a V_2. This last means that we assume the rank of the first fundamental tensor a'_{cb} of the V_2 is two. On the V_2, we introduce two orthogonal non-isotropic

Received March 21, 1938, and July 1, 1938; in revised form, December 3, 1938.

[1] Schouten and Struik, *Einführung in die neueren Methoden der Differentialgeometrie*, Batavia, vol. II, 1938, p. 108. We shall refer to this volume as II.

[2] II, p. 99.

[3] I am deeply indebted to Professor D. J. Struik for his aid in revising part A of this paper.