INTERIOR TRANSFORMATIONS ON CERTAIN CURVES

BY G. T. WHYBURN

In this paper a study will be made of interior transformations as applied to compact metric continua. Also results will be established concerning such transformations defined on certain particular classes of curves, such as dendrites and boundary curves. A single-valued continuous transformation $T(A) = B$ is said to be interior [2][1] provided the image of every open set in A is a set open in B. All continua referred to in this paper are assumed to be compact; and if R is an open set, the boundary of R, i.e., the set $\bar{R} - R$, is designated by $F(R)$.

1. **Conditions for lightness.** A transformation $T(A) = B$ is said to be *light* [5] provided that for no $b \epsilon B$ does $T^{-1}(b)$ contain a non-degenerate continuum. Inasmuch as the property of being light is assumed by Stoïlow [2] for all interior transformations, it is of interest to determine certain classes of continua on which all interior transformations are necessarily light.

(1.1) THEOREM. *If A is a locally connected continuum such that the boundary of every region in A is totally disconnected, then every interior transformation which does not carry A into a single point is light.*

Proof. Suppose, on the contrary, that there exists an interior transformation $T(A) = B$ and a point $p \epsilon B$ such that $T^{-1}(p)$ contains a non-degenerate continuum H. Let R be a component of $B - p$ and let $R_1, R_2, \cdots, R_n$ be the components of $T^{-1}(R)$. Then since T is interior, we have

$$H \subset F(R_1) + F(R_2) + \cdots + F(R_n).$$

Accordingly, for some $i \leqq n$, $F(R_i) \cdot H$ must contain a non-degenerate continuum, contrary to hypothesis.

(1.11) COROLLARY. *If A is either* (i) *a dendrite,* (ii) *a locally connected continuum no cyclic element of which has a continuum of condensation, or* (iii) *a continuum every subcontinuum of which contains uncountably many local separating points of A, then every interior transformation on A is light.*

(1.2) THEOREM. *In order that a continuous transformation $T(A) = B$ be light (where A is compact), it is necessary and sufficient that for any $\epsilon > 0$ a $\delta > 0$ exists such that if X is any continuum in B of diameter $< \delta$, each component of $T^{-1}(X)$ is of diameter $< \epsilon$.*

Proof. The sufficiency is immediate. For if we suppose the condition satisfied and that there is a non-degenerate component H of $T^{-1}(p)$ for some $p \epsilon B$,

Received May 30, 1938.

[1] The numbers in brackets refer to the bibliography at the end of the paper.