RELATIONS BETWEEN CERTAIN CONTINUOUS TRANSFORMATIONS OF SETS

BY R. G. SIMOND

In a paper on arc-preserving transformations[1] G. T. Whyburn has shown that under certain conditions a transformation is a homeomorphism on each cyclic element of a compact locally connected continuum. The principal result is obtained by using a transformation which is arc-preserving and irreducible. After studying Whyburn's theorems, the writer investigated the problem of finding conditions for a homeomorphism on the whole of any compact locally connected continuum and also of finding other conditions for a homeomorphism on the cyclic elements. In working with continua other than cyclic elements, it was found necessary to define some new transformations and to impose conditions on the sets obtained by the transformations. After true arc-preserving and strongly irreducible transformations were defined, it was found that one of Whyburn's proofs could be modified to give a proof of conditions for a homeomorphism on any compact locally connected continuum. The writer has also defined a tree-preserving transformation and has found additional conditions which will make it a homeomorphism in both cases, first on the cyclic elements and then on the whole of any compact locally connected continuum. To do this, true tree-preserving and strongly monotonic transformations have been defined. After these various transformations were studied, it was found that certain relations existed between them, and theorems about some of these relations are also proved in this paper.

The writer wishes to express her appreciation for the help given by Professor W. L. Ayres of the University of Michigan, who suggested the investigation of this problem and gave valuable criticism and advice during the preparation of this paper.

We shall assume throughout that our space is metric and compact. A knowledge of the theory of cyclic elements will also be assumed. Definitions of and theorems about cyclic elements may be found in an expository paper by Kuratowski and Whyburn.[2]

The symbol $\Phi(A)$ will mean a transformation Φ on the set A, where A is a compact locally connected continuum. All of the transformations which are considered are assumed to be single-valued and continuous. For a single-valued transformation continuity is equivalent to the property that a closed set comes

Received March 31, 1938; presented to the American Mathematical Society, April 9, 1937.

[1] G. T. Whyburn, American Journal of Mathematics, vol. 58(1936), pp. 305–312.

[2] C. Kuratowski and G. T. Whyburn, Fundamenta Mathematicae, vol. 16(1930), pp. 305–331.