

**SOLUTIONS OF SYSTEMS OF DIFFERENTIAL EQUATIONS IN  
INFINITELY MANY UNKNOWN BY INFINITE SERIES OF  
DEFINITE INTEGRALS**

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**Introduction.** The method used in two papers by the author<sup>1,2</sup> can be extended to systems of differential equations in infinitely many dependent variables, but the integrands of the definite integrals appearing in the solutions consist of an infinite number of terms. F. R. Moulton has also examined the case where the right members of the differential equations

$$(1) \quad \frac{dx_i}{dt} = f_i(t, x_j) \quad (i, j = 1, 2, \dots)$$

are analytic in the variables  $t$  and  $x_j$  and  $f_i(0, 0, \dots) = 0$ .<sup>3</sup>

In all three of these methods the solutions are given in terms of infinite series each term of which is an infinite series. The methods are theoretically correct, but as each term depends upon the preceding terms, it is clear that a serious practical obstacle is encountered when the terms after the first ones are to be computed.

Other known methods of approach to the problem are general analysis and infinite matrices.

In the present paper the solutions are given in terms of infinite series of definite integrals each integrand of which has a finite number of terms.

The system of differential equations to be considered has the form

$$(2) \quad \frac{dx_i}{dt} = \theta_i(t) + \sum_{h=1}^{\infty} f_{i\mu_1 \dots \mu_\nu}(t) x_1^{\mu_1} x_2^{\mu_2} \dots x_\nu^{\mu_\nu} \quad (i, \nu = 1, 2, \dots),$$

where  $\mu_1, \dots, \mu_\nu$  are non-negative integers and

$$(3) \quad h = \mu_1 + 2\mu_2 + \dots + \nu\mu_\nu,$$

for all positive integral values of  $\nu$ . The above arrangement of the terms in the right members of (2) is made in order that every term with a finite number of the  $x$ , with finite exponents will appear after a finite number of terms. The path of integration for the definite integrals appearing in the solution functions is the interval  $(t_0, t)$ , where the variable  $t$  is real and

$$(4) \quad |t_0 - t| = \mu.$$

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<sup>1</sup> This Journal, vol. 3(1937), pp. 616-622.

<sup>2</sup> American Mathematical Monthly, vol. 43(1936), pp. 530-539.

<sup>3</sup> F. R. Moulton, *Differential Equations*, p. 375.